Hair Animation with Collision Detection

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ABSTRACT

We propose an efficient method for hair animation. The motion of hairs is modeled by a simplified physical simulation. Collision between hair and a human body or other objects is used to make a realistic hair animation. Fast collision detection is required because the number of hairs is very large. Therefore, a cylindrical representation of human body or other objects is created for efficient collision detection. Because of the cylindrical representation, collision is detected with table look-up and interpolation, and computation complexity is independent of the complexity of the body. Reaction constraint method is applied for the collision response to simulate inelastic contact. This method is successfully applied to generate hair animation.

Keywords : human character animation, hair, collision detection.

1. INTRODUCTION

Human character animation is one of the most challenging topics in computer graphics (Magnenat-Thalmann and Thalmann 1991). Human hair is important to generate natural human characters. However, there remain many problems to overcome for hair animation, because the number of hairs is very large (hundreds of thousands of strands).

Several techniques have been proposed relating to hair rendering. Miller (1988) successfully generated furry object by modeling each strand as long triangles. Kajiya and Kay (1988) used three dimensional texture mapping, *texel*, to generate images of teddy bear. LeBlanc et al. (1992) used antialiasing and shadow buffer techniques to generate naturalistic images of human hair. Rosenblum et al. (1992) also used similar techniques. Watanabe and Suenaga (1992) proposed a 'trigonal prism and wisp model' to represent human hair.

These successful results of hair rendering have shown that more realistic human characters can be synthesized with modeling and animation techniques of hair. However, research of modeling and animation techniques is still an open field.

The movement of hair is governed by physical law, so physically based modeling approach (Terzopoulos and Fleischer 1988) may make good results. Animating single strand is not difficult problem using physically based approach. However, the number of hair makes it impossible or impractical to apply precise physical simulation to the movement of hair. Therefore, methods to animate hair assuming some simplification have been proposed. Rosenblum et al. (1992) proposed an animation method of hair using a mass spring model. Anjyo et al. (1992) have proposed methods using one dimensional projective differential equations and pseudo-force field. Both methods neglect the effect of collision between hairs for simplicity. The collision of hair with other objects, such as the head, is simplified using a sphere or ellipsoid. The problem of self collision of hair is very difficult. However, the collision between hair and other objects is required to generate naturalistic movement of long hair.

Collision detection is an important issue in physically based approach, and several methods have been proposed (Platt and Barr 1988, Moore and Wilhelm 1988, Lafleur et al. 1991). Again, the number of hairs makes it impractical to apply these methods precisely. Ordinary methods of collision detection by polygon and line intersection algorithms are too expensive to generate hair animation within the limitation of current computer technology. A simplified approach to collision detection is also required as is the physical simulation of hair.

This paper presents a simplified and efficient method for hair animation with collision detection. Section 2 describes a dynamic model of hair based on simple differential equations of one-dimensional angular momenta. Section 3 introduces an efficient collision detection method using a cylindrical representation. Section 4 shows several examples of hair animation. Section 5 discusses advantage and limitation of the method.

2. DYNAMIC MODEL OF HAIR

We follow the dynamic model of hair proposed in (Anjyo et al. 1992). Although, a hinge effect is added for representing wavy hair, and a damping effect is added for more naturalistic movement. A strand of hair is modeled as a series of line segments (Figure 1). In this figure, s_i ($1 \le i \le k$) is segment, P_i ($0 \le i \le k$) is node, d is the length of each segment, and k is the number of segment in each strand. If the strand is not stretched, the shape of the hair is represented by the angles between segments. Taking the polar coordinate system as shown in Figure 2, the behavior of the zenith angle θ_i and the azimuth ϕ_i of segment s_i are observed. This polar coordinate system is defined according to the coordinate system of the parent segment.

The variables $\theta_i(t)$ and $\phi_i(t)$ with the time parameter t are governed by the ordinary differential equations:

$$I_i \frac{d^2\theta_i}{dt^2} + \gamma_i \frac{d\theta_i}{dt} = M_\theta$$
$$I_i \frac{d^2\phi_i}{dt^2} + \gamma_i \frac{d\phi_i}{dt} = M_\phi, \qquad (1)$$

where I_i is the moment of inertia of the segment s_i, γ_i is the damping coefficient, M_θ and M_ϕ are the torque according to θ and ϕ component respectively.

The inertial moment of segment s_i is defined as

$$I_i = \frac{1}{3\,i}\lambda k^4 d^3 , \tag{2}$$

where λ is the line density. The torque M_θ and M_ϕ applied to the segment s_i are derived from the hinge effect $M_{\theta spring}$, $M_{\phi spring}$ between two segments, and external moment $M_{\theta external}$, $M_{\phi external}$ from external force, such as gravity, inertial force and wind:

$$\begin{aligned} M_\theta &= M_{\theta\ spring} + M_{\theta\ external} \\ M_\phi &= M_{\phi\ spring} + M_{\phi\ external} \end{aligned} \;. \tag{3}$$

$M_{\theta spring}$ and $M_{\phi spring}$ are defined as

$$\begin{aligned} M_{\theta\ spring} &= -k_\theta(\theta - \theta_0) \\ M_{\phi\ spring} &= -k_\phi(\phi - \phi_0) \end{aligned} , \tag{4}$$

where k_θ and k_ϕ are spring constants, and θ_0 and ϕ_0 are the initial angles.
External moments are defined as

$$\begin{aligned} M_{\theta\ external} &= u\, F_\theta \\ M_{\phi\ external} &= v\, F_\phi \end{aligned} , \tag{5}$$

where u is (1/2)d, v is the half length of the segment that is the projection of s_i onto the ϕ plane (see Figure 3 and 4). F_θ, F_ϕ are the "θ, ϕ -component" of force **F** respectively.

The above θ component F_θ of the applied force **F** is the scalar value defined by $F_\theta = (\mathbf{F}, \mathbf{V}_\theta)$, where $\mathbf{V}_\theta$ is the unit vector on the θ plane that is perpendicular to the segment s_i. Similarly, the ϕ component F_ϕ is defined by $F_\phi = (\mathbf{F}, \mathbf{V}_\phi)$, where $\mathbf{V}_\phi$ is the unit vector on the ϕ plane that is perpendicular to the projected segment of s_i onto the ϕ plane.

The external force **F** is defined as

$$\mathbf{F} = \rho d\,(\mathbf{g} + \mathbf{a}) + d\,\mathbf{f}, \tag{6}$$

where **g** is the acceleration due to gravity, **a** is the acceleration due to the movement of the head itself, and **f** is the density of the applied force, such as wind.

In the numerical simulation, equation (1) is discritized as

$$\theta_i^{n+1} - 2\theta_i^n + \theta_i^{n-1} + \gamma_i \Delta t\, (\theta_i^n - \theta_i^{n-1}) = (\Delta t)^2\, M_\theta$$
$$\phi_i^{n+1} - 2\phi_i^n + \phi_i^{n-1} + \gamma_i \Delta t\, (\phi_i^n - \phi_i^{n-1}) = (\Delta t)^2\, M_\phi \quad . \qquad (7)$$

The calculation starts with the segment s_1, and the new angle of s_i is successively determined using (7).

3. COLLISION DETECTION AND RESPONSE

In order to generate naturalistic hair animation, collision between hair and other objects must be considered. Ordinary collision detection methods find intersections between polygons and points. However, this method is impractical because of the number of hairs, and the number of polygons of human body or other objects. It is desirable to apply an efficient collision detection method whose complexity is independent of the complexity of objects. In this section, we propose a simplified and efficient collision detection method for hair animation. This method utilizes a cylindrical representation.

3.1 Collision Detection using Cylindrical Projection

Collision detection in our case is to determine whether strand segment is inside of other objects or not. A correct test must consider line segments and objects. However, we consider only node point and objects test for simplicity. Then the problem is to determine whether node **P** is inside of the objects or not.

First, we create a cylindrical representation of human body or other objects. The human body is very irregular and cannot be described as one cylindrical representation. Therefore, we divide human body or other objects into several parts, and each part is described with cylindrical representation. For example, the human body is divided into a head, trunk, arms, and legs. Each part is described in a cylindrical coordinate system as (r, θ, y), where r is radius, θ is azimuth, and y is height. We prepare an array of radius on discretized azimuth $i\Delta\theta$ and height $j\Delta y$ $(1 \leq i \leq m,\ 1 \leq j \leq n)$ by calculating intersection between line and object (Figure 5). Along with the radius, we prepare the normal vector on each sample point for calculating collision response. Figure 6 shows examples of a cylindrical representation of the human body. Figure 6a is the original human body model with 7000 polygons. Figure 6b shows head and trunk model represented by cylindrical coordinate system. These two models are defined with [40 x 40] array of radius values.

When a cylindrical representation is created, collision is easily detected. Suppose a node point **P** is tested whether it is inside of the object of not. First, we translate point **P** into cylindrical coordinates as (r_p, θ_p, y_p). Second, we obtain the corresponding point **Q** of the object with the same azimuth θ_p and height y_p as point **P**. If we assume the smoothness of object surface, position of **Q** can be approximated by linear interpolation as follows. We find index (i, j) which satisfies

$$i\,\Delta\theta \leq \theta_p < (i+1)\Delta\theta$$
$$j\,\Delta y \leq y_p < (j+1)\Delta y \quad . \qquad (8)$$

Radius of point **Q** is then approximated as: