

# Scalable Visual Hierarchy Exploration

by

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*Cu dragoste, parintilor mei.*

## Abstract

More and more modern computer applications, from business decision support to scientific data analysis, utilize visualization techniques to support exploratory activities. Most visualization tools do not scale well with regard to the size of the dataset upon which they operate. Specifically, the level of cluttering on the screen is typically unacceptable and the performance is poor. To solve the problem of cluttering at the interface level, visualization tools have recently been extended to support hierarchical views of the data, with support for focusing and drilling-down using interactive brushes. To solve the scalability problem, this thesis investigates how best to couple a visualization tool with a database management system without losing the real-time responsiveness required in any interactive application.

This integration must be done carefully. Visual user interactions often cannot be efficiently realized using standard database operations. In our context, the recursive processing required for the visual data retrieval was prohibitively expensive; responding for instance to a single user request took up to 30 minutes on a 200,000 tuple dataset. We successfully address this problem by developing a tree labeling method, called MinMax tree, that moves the recursive processing into an off-line precomputation step. Thus, at run time, the recursive operations translate into linear cost range queries. To further reduce the response time, we employ a main memory access strategy to support incremental loading of data into main memory. We also proposed a novel prefetching technique to bring data into memory when the system is idle. Prefetching is speculative, non-pure, and adaptive. These techniques have been incorporated into XmdvTool, a multidimensional visual exploration tool, in order to achieve scalability. The tool now efficiently scales up to datasets of order  $10^5 - 10^7$  records. Lastly, we report experimental results to assess the performance of the proposed techniques.

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# Chapter 1

## Introduction

### 1.1 Caching and Prefetching for Large-Scale Visualization

Visualization provides effective techniques for the analysis of data. While statistics offers us various tools for testing model hypotheses and finding model parameters, the task of guessing the right type of model to use is still a process that cannot be automated. Thus, whether the domain is stock data, scientific data, or the distribution of sales, visualization plays an important role in the analysis. Humans can sometimes detect patterns and trends in the underlying data by just looking at it, *without* being aware in advance about what data model they'll face.

Human perception is of course greatly influenced by the way data is presented. Thus, various techniques for displaying data have been proposed over the years, each of which focuses on emphasizing some of the data characteristics [1, 26, 8, 14, 30]. However, most of these techniques do not scale well with respect to the size of the data. As a generalization, [17] postulates that any method that displays a single entity per data point invariably results in overlapped elements and a convoluted display that is not suited for

the visualization of large datasets.

A new approach has been proposed recently for displaying large datasets [16]. The idea there is to present data at different levels of detail based on applying an aggregation function to a hierarchical structure, structure that might result for instance from a clustering process. The problem of cluttering at the interface level is solved by displaying only a limited set of aggregates at a time. However, such hierarchical summarizations increase the size of data to be managed by at least one order of magnitude.

Storing and retrieving the data sets efficiently has often been ignored in the context of visualization applications. While storing the data in main memory and flat files is appropriate for small and moderate sized datasets, it becomes unacceptable when scaling to large datasets. One possible solution to enable scaling is to integrate visualization applications with database management systems. The last couple of decades of research in the area of databases can greatly contribute to increasing the performance of a data intensive application such as exploratory visualization.

Coupling a database with the visualization tool cannot be performed *blindly* though. Techniques used for main memory processing are typically not efficient any more if implemented directly in a database environment. A trivial and well known example is sorting. Internal sorting strategies differ significantly from external sorting ones. Another example is presented in this work: the recursive processing involved when navigating through hierarchies in main memory is no longer appropriate when storing those hierarchies on the disk. Instead, we propose a technique called *MinMax trees* [40, 41] that transforms the recursive processing into fast range queries.

In general, there are two questions to be answered when doing such an integration between visualization and database tools. The first is how to most effectively translate the visual exploration operations such as zooming and brushing into a database understandable language such as SQL. The second is how to store and manage the results of

the database requests in main memory and make subsequent memory access operations as efficient as possible. This work gives solutions for both questions.

## 1.2 Our Approach

Our approach to reducing the system latency (i.e., the on-line computation time) is to push expensive computations off-line whenever possible. Visual tools require data to be in the same address space to access it. In a simple main memory system, the data resides entirely in the main memory (Fig. 1.1). When the visualization tool issues a request, the entire computation is performed “on-demand” (i.e., on-line). The result is a set of objects (datapoints) that is then passed back to the front-end. When the data is moved from the main memory to persistent storage, two additional steps have to be present (Fig. 1.2). First, the request needs to be translated into a format which is supported by the query processor and then passed to the database for processing (also on-line). Second, the result set needs to be loaded into the main memory and sent to the front-end for display. The “on-demand” computation as well as data loading are I/O intensive and thus no longer fast (compared to the main memory case). Our goal of minimizing the system latency can be achieved by optimizing these two steps.

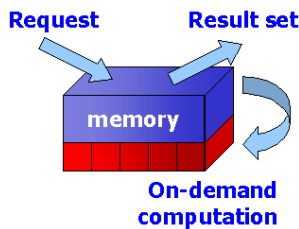


Figure 1.1: Architecture of main memory-based implementation.

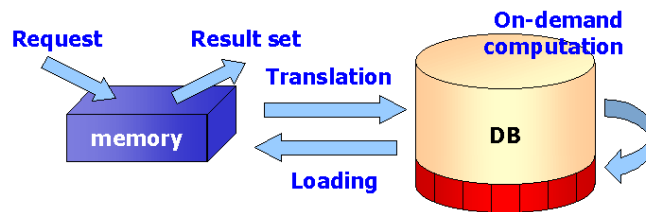


Figure 1.2: Architecture of database-based implementation. Additional computation steps are I/O intensive.

Our approach to making the first step efficient is to move part of the on-line compu-

tation into a pre-processing phase. In our case, the front-end operations are reducible to a particular class of recursive unions of joins and divisions that operate on hierarchies. By using adequate pre-computation (i.e., organizing the hierarchical structure into what we call a MinMax tree), the recursive processing of the operations in this class can be reduced to range queries. Extensions of our method for non-tree as well as for dynamic hierarchies have also been designed (Chapter 4). Compared with alternate approaches to similar problems from the literature [7, 44], our hierarchy labeling method is shown to be superior in terms of both efficiency and functionality. The previous proposed methods either do not support dynamic and arbitrary hierarchies, or were not able to efficiently scale to large datasets.

To make the second step efficient, we employ a main memory strategy that supports incremental loading of data into the main memory. We show that incremental loading is highly desirable, given the set of operations to support (Chapter 5). To further reduce the response time, we have designed a speculative prefetcher that brings data into memory when the system is idle. The prefetcher is based on the property of exploratory systems that queries remains “local”, i.e., given the set of currently selected objects we have a small number of choices of which objects can be selected next. The property provides therefore “implicit hints” to the system. Additional hints might be provided by the the data and the user’s exploration as well.

## **1.3 Contributions**

Our main contribution consists of developing a set of techniques that can be applied to interactive visualization tools in order to enable them to explore large datasets. The characteristics of the system to which these techniques can be applied to are presented in Chapter 2.



We first designed and implemented an encoding technique that efficiently supports on-line retrieval of data. We further designed a high level cache policy that reduces the latency in the system by incrementally loading the data into the memory buffer. When the system is idle, a prefetcher will bring into cache the data that is likely to be used next. For this purpose, we used a novel technique that combines a low granularity of data (object level) and a “semantic” description of the content of the buffer. We also performed experiments to assess the efficiency of our approach. First, we tested our encoding technique as a stand-alone method and found that it performed and scaled well. Second, we tested the system to work with various input and under different settings. The system scales well to support millions of data points with between 8 and 20 dimensions. We confirmed the important role of precomputation in such applications and showed that the benefit of using prefetching exceeds significantly the result from using the cache only.

## **1.4 Thesis Organization**

Chapter 2 presents the related research from both the visualization and the database perspective. Chapter 3 introduces basic concepts in visualization and describes our approach to achieve scalability from a front-end perspective. It also formalizes the requirements under which our proposed approach works. The hierarchy encoding as well as the processing of MinMax queries is further presented in Chapter 4. Chapter 5 introduces the memory management, specifically the caching and prefetching strategies. The implementation of the system is discussed in Chapter 6. Experimental results are reported in Chapter 7. Finally, we present conclusions as well as directions for further research in Chapter 8.

# Chapter 2

## Related Work

### 2.1 Visual Hierarchy Exploration

There has been considerable research in the visualization area toward finding effective methods to display and explore hierarchical information, such as Tree-Maps [37], Cone-Trees [32] and Reconfigurable Disc Trees [22]. Most of these methods provide only modest modes of interaction for navigating the hierarchy. Navigation plays an important role in aiding users to find their way through the complex structure: to see where they are, what information is available and how to identify information of interest.

On the other hand, techniques for visual exploration of hierarchies [27] have independently been proposed. Hierarchy visualizations are evident for instance in many commercial applications, such as Microsoft Windows Explorer, Norton Commander, and so on. The major disadvantage of such interfaces however is that there is a limited display space for the hierarchy. Hence, they are not suitable for displaying large data sets. The visualization technique we use in this work [16, 17] has both the capability of interactively navigating the hierarchical structures and the capability of displaying large datasets.

## 2.2 Visualization-Database Integrated Systems

Integrated visualization-database systems such as Tioga [39], IDEA [34], and DEVise [28] represent the work closest related to ours in terms of problem area. The approaches are however different. Tioga [39] implements a multiple browser architecture for what they call a *recipe*, a visual query. The system is able to buffer the computed data; however, the problem of translating front-end operations into database queries is not present since database queries are directly (explicitly) specified by the graphical interface. IDEA [34] is an integrated set of tools to support interactive data analysis and exploration. Some constraints on the data model are imposed by the application domain, but on-line query translation and memory management are not addressed. In DEVise [28], a set of query and visualization primitives to support data analysis is provided. The number of primitives supported is itself relatively large. However, caching data is done at the database level using default mechanisms only; special memory management techniques are not considered.

Other work in the same area includes dynamic query interfaces [43, 21], dynamic query histograms [13] and direct manipulation query interfaces [20, 24, 19]. They all have a visual interface and a database back-end. However, the operations translate differently: to dynamic range queries in [13], to temporal queries in [20], and to 2-D spatial queries in [24]. These works do not deal with hierarchy exploration support.

## 2.3 Relational Processing of Hierarchies

### 2.3.1 Join Processing

In relational systems, hierarchies (as composite objects) have to be broken down into multiple fragments that are then stored as tuples in separate relations [46]. Traversing the

hierarchical structure in order to gather all fragments together or to query specific properties requires a large number of joins. Since relational joins are expensive operations, an immediate improvement in handling hierarchical structures is achieved by improving join efficiency.

Valduriez et al. [45] introduce a new access path for processing joins, called a *join index*. The join index is simply a binary relation that contains pairs of surrogates (unique system identifiers) of the tuples that are to be joined. An algorithm that uses join indices is also presented in [45]. The join index efficiently supports the computation of joins and particularly the join composition of complex objects in the case of a decomposed storage representation [46].

Another method that speeds up join processing uses hidden pointer fields to link the tuples to be joined. The hidden pointers are special attributes that contain record identifiers. Three pointer-based join algorithms, simple variants of the nested-loops, sort-merge and hybrid-hash join, are presented and analyzed in [36].

A hash-based method for large main memory systems is described in [35]. The author concentrates on the improvement of joins based on the traditional strategy of sort and merge. Three algorithms are evaluated: a simple hash, the GRACE hash from the 5th Generation Systems, and a hybrid of the two. When the available main memory exceeds at least the square root of the size of one relation, the hash-based algorithms can successfully be applied for computing joins. Their gain is especially significant when large relations are involved.

The above techniques make join processing efficient, but they don't limit in any way the recursive processing typically involved when traversing hierarchies. The number of system calls is high and many intermediate tuples are unnecessarily retrieved.

### 2.3.2 Hierarchy Encoding

Another way to handle hierarchies has emerged with the development of object-relational systems [38]. Using object extensions a composite object can be represented using nested (non 1-st NF) relations. However, recursive relations do not always have a pre-defined depth and therefore they cannot be represented using nesting.

A novel idea in hierarchical processing was introduced by Ciaccia et al. [7]. They encode tree hierarchies based on the mathematical properties of *simple continued fractions* (SICFs). Basically, each node of the tree has a unique label that encodes the ancestor path from that node up to the root. The trees are assumed to be ordered (i.e., children have order numbers) so that the ancestor path simply corresponds to a sequence of integers. The sequence gives us the code of the ancestors of a node without any physical access to the data. This information is sufficient for performing some operations, such as getting the first common ancestor of two nodes or testing if a node is the ancestor of another one, without any recursive retrieval of data. However, given a node  $n$ , this method cannot, for example, efficiently provide the list of descendants of  $n$ . This limitation reduces the number of operations that can be supported and, moreover, makes updates difficult to handle. Another important limitation of this method is that it can only be applied to tree hierarchies and not to arbitrary hierarchies.

A similar idea was introduced by Teuhola in [44]. He used a so called *signature* for encoding the ancestor path. The important difference of the signature method to the previous approach is that now the code is not unique. Given a node  $n$ , the code of  $n$  is obtained by applying a hash function to it and by concatenating the resulting value with the code of its parent. The non-unique code can make the quantity of data retrieved be much larger than needed. Moreover, the code obtained by the concatenation of all ancestor codes could exceed the available precision for deep trees. A fragmentation of the initial tree and consequently additional joins would thus need to be performed.

## 2.4 Main Memory Processing

### 2.4.1 High Level Caching

High level caching systems in which objects are not individually identified, but rather a set of objects together is identified with the query that generated it, is called *semantic caching* [12] or *predicate caching* [25]. Our memory management is similar to the one present in semantic caching. The buffer content is specified by a set of queries. However, due to the specific requirements we had and for efficiency, we applied the concepts of semantic caching quite different, enabling data to be handled also at a smaller granularity (i.e., at the object level). Other work in the area of object level caching for database applications have been addressed for example in [9, 33]. Also, object based caching has been studied recently in the context of web applications [15].

### 2.4.2 Prefetching

In many interactive database applications, there is often sufficient time between user requests, and therefore the amount of data that can be prefetched is limited only by the cache size. This situation is referred to as *pure prefetching* and constitutes an important theoretical model in analyzing the benefit of prefetching. In practice however prefetch requests are often interrupted by user requests, resulting in less data being prefetched at a time. In such cases, called *non-pure prefetching*, issues of cache replacement also need to be considered. Pure prefetchers can be converted into practical non-pure ones by combining them with a cache replacement strategy. In [11] for instance, a pure prefetcher is used with the least recently used (LRU) cache replacement strategy, and a significant reduction in the page fault rate was shown. A multi-threaded implementation of a non-pure prefetcher is reported in [42]. There, the latency of the disk operations is improved by using threads.

The estimation strategy, called also a *predictor*, is usually based on either a probabilistic model or some recorded statistics [5]. A widely used predictor in systems similar to ours is based on Markov chain theory [2, 23]. The main idea is that given a string  $s = (\alpha_{i_1}, \alpha_{i_2}, \dots, \alpha_{i_n})$  of letters over an alphabet  $\Sigma = (\alpha_i)_i$ , we can compute the probability of letter  $\alpha_j$  being at position  $n + 1$  based on the patterns existing in  $s$ . Markov predictors have been first used in prefetching in the context of paged virtual memory systems [2] under the name *correlation-based prefetching*. [23] also uses Markov predictors for prefetching and reports good results.

# Chapter 3

## Multivariate Data Visualization

The work presented in this paper was triggered by our goal of scaling *XmdvTool* to work on large data [47]. *XmdvTool* is a software package designed for the exploration of multivariate data. The tool provides four distinct visualization techniques (scatterplot matrices, parallel coordinates, glyphs, and dimensional stacking) with interactive selection and linked views. Recent efforts have produced hierarchical parallel coordinates, that allow multi-resolution data presentation [16]. The main idea is to cluster the datapoints based on a distance metric, apply an aggregation function to the datapoints from each cluster and have those aggregate values displayed instead of the datapoints themselves. The model can be conceptualized as a hierarchy that provides the capability of visualizing data at various *levels of abstraction*. The hierarchical structure can be explored by interactively selecting and displaying points at different levels of detail. We term this exploration process *navigation*. In what follows we describe these visual exploration operations in more detail and then provide a formal model that summarizes the semantics of these operations.



### 3.1 Brush Basics

*Selection* is a process whereby a subset of entities on a display is isolated for further manipulation, such as highlighting, deleting, or analysis. Wills [48] defined a taxonomy of selection operations, classifying techniques based on whether memory of previous selections is maintained or not, whether the selection is controlled by the underlying data or not, and what specific interactive tool (e.g., brushing, lassoing) is used to differentiate an area of the display. He also created a selection calculus that enumerates all possible combinations of actions between a previous selection and a new selection (replace, add, subtract, intersect, and toggle) and attempted to identify configurations of these actions that would be most useful.

*Brushing* is the process of interactively painting over a subregion of the data display using a mouse, stylus, or other input device that enables the specification of location attributes. The principles of brushing were first explored by Becker and Cleveland [3] and applied to high dimensional scatterplots. Ward and Martin [47, 29] extended brushing to permit brushes to have the same dimensionality as the data ( $N$ -D instead of 2-D). They also explored the concepts of multiple brushes, composite brushes (formed by logical combinations of brushes), and *fuzzy* brushes, that allow points to be partially contained within a brush. Haslett et al. [18] introduced the ability to show the average value of the points that are currently selected by the brush.

One common method of classifying brushing techniques is to identify in which space the selection is being performed, namely screen or data space. This can then be used to specify a *containment criterion* (whether a particular point is inside or outside the brush). In *screen space* techniques, a brush is completely specified by a 2-D contiguous subspace on the screen. In *data space* techniques, a complete specification consists of either an enumeration of the data elements contained within the brush or the  $N$ -D boundaries of a

hyper-box that encapsulates the selection.

A third category, namely *structure space* techniques, which allows selection based on structural relationships between data points, has been introduced in [17]. The *structure* of a data set specifies relationships between data points. This structure may be explicit (e.g., categorical groupings or time-based orderings) or implicit (e.g., resulting from analytic clustering or partitioning algorithms). Examples of structures include linear orderings, tree hierarchies, and directed acyclic graphs (arbitrary hierarchies). In this work we focus on tree hierarchies. A *tree* is a convenient mechanism for organizing large data sets. By recursively clustering or partitioning data into related groups and identifying suitable summarizations for each cluster, we can examine the data set methodically at different levels of abstraction, moving down the hierarchy (*drill-down*) when interesting features appear in the summarizations and up the hierarchy (*roll-up*) after sufficient information has been gleaned from a particular subtree.

As described earlier, brushing requires some containment criteria. For our first containment criterion, we augment each node in the hierarchy, that is each cluster, with a monotonic value relative to its parent. This value can be, for example, the level number, the cluster size/population, or the volume of the cluster (defined by the minimum and maximum values of the nodes in the cluster). This assigned value determines the control for the level-of-detail. Our second containment criterion for structure-based brushing is based on the fact that each node in a tree has extents, denoted by the left- and right-most leaf nodes originating from the node. In particular, it is always possible to draw a tree in such a way that all its children are horizontally ordered. These extents ensure that a selected subspace is contiguous in structure space.

A structure-based brush is thus defined by a subrange of the structure extents and level-of-detail values. Intuitively, if looking at a tree structure from the point-of-view of its root node (Fig. 3.1), the extent subrange appears as a *focus region* (with the focus point

at its center), while the level-of-detail subrange corresponds to a sampling rate factor or a *density*. In a 2-D representation of the tree (Fig. 3.2), the subranges correspond to a horizontal and vertical selection, respectively.

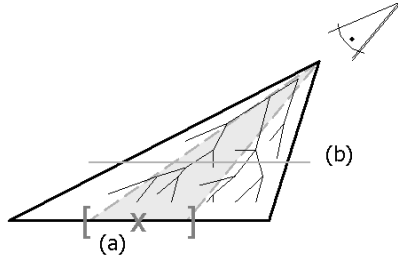


Figure 3.1: Structure-based brush as combination of a *focus region* (a) and a *density factor* (b).

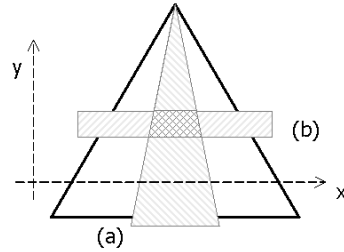


Figure 3.2: Structure-based brush as combination of a horizontal (a) and a vertical (b) selection.

## 3.2 Hierarchical Clustering

In what follows, we describe the clustering process used to organize the data in Xmdv-Tool. The clustering phase generates the hierarchical tree which is further used during exploration, but is not a pre-requisite for our technique. Any other method that generates a similar data structure may be used as well.

Let  $X$  be a data set composed of  $m$  data points. The elements of  $X$  are called *base data points*. A hierarchical clustering is obtained by recursively aggregating elements of  $X$  into intermediate groups (clusters). Conceptually, the hierarchical clustering can be thought as an iterative process of successive cluster aggregations that starts with the elements of  $X$  ( $m$  clusters of one element each) and ends with a large cluster that incorporates all the elements of  $X$ . A state of this transitory process can be defined as a partition on the elements of  $X$ . The next state is also a partition obtained by grouping some of the sub-sets of the previous partition. Two such successive partitions are called *nested*. Consequently, we can define a hierarchical clustering of  $X$  as a sequence of nested partitions, in which

the first one is the trivial partition and the last one is the set itself. A formal definition of hierarchical clustering is presented in Appendix A.

A graphical representation of an example of hierarchical clustering is presented in Fig. 3.3 for a set of five elements  $\{a, b, c, d, e\}$ . We call this representation a *partition map*.

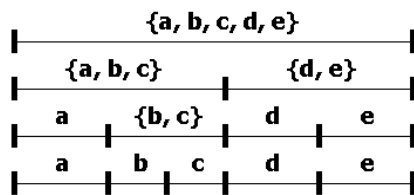


Figure 3.3: Partition map for a tree hierarchy.

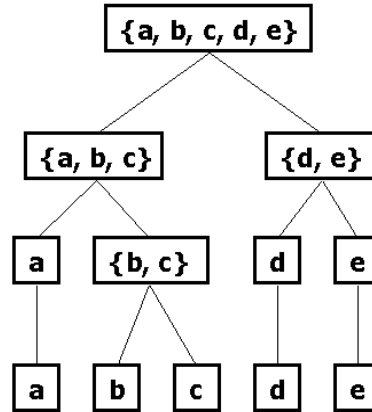


Figure 3.4: Hierarchical tree obtained by clustering.

A hierarchical clustering may also be organized as a tree structure  $T$ , where the root is the whole  $X$  and the leaves are base data points. A node of  $T$  corresponds to an aggregation whenever it has more than one child. A graphical representation of such a cluster tree, obtained by hierarchical clustering of the same set of five elements, is presented in Fig. 3.4.

Data can be hierarchically structured either explicitly, based on explicit partitions (such as, for example, in category-driven partitioning) or implicitly, based on the intrinsic values of the data points. In the latter case a clustering algorithm needs to be used to form the hierarchy. We have tried two clustering algorithms in our system, but others would be suitable as well. Specifically, we have used BIRCH [50] as well as a simple one of ours

[49].

## 3.3 Structure-Based Brushes

### 3.3.1 Creation and Manipulation

Figure 3.5 shows the structure-based brushing interface implemented in XmdvTool [17]. The triangular frame depicts the hierarchical tree. The contour near the bottom of the frame delineates the approximate shape formed by chaining the leaf nodes. The colored bold contour across the tree delineates the tree cut that represents the cluster partition corresponding to the specified level-of-detail. XmdvTool uses a *proximity-based coloring scheme* in assigning colors to the partition nodes [16]. In this scheme, a linear order is imposed on the data clusters gathered for display at a given level-of-detail. This linear order is directly derived from the order in which the tree is traversed when gathering the relevant nodes for a given level-of-detail. Colors are then assigned to each cluster by looking up a linear colormap table. The same colors are used for the display of the nodes in the corresponding data display. The two movable handles on the base of the triangle, together with the apex of the triangle, form a wedge in the hierarchical space.

Since, as shown before, a structure-based brush is defined as the intersection of two independent selections, it necessarily follows that setting such a brush requires two computational phases as well. The first one, the horizontal selection, is accomplished in two steps. In the first step a set of leaf nodes is initially selected based on the order property using the two handles indicated by  $e$  in Fig. 3.5. Basically, this step corresponds to “select all leaves between the two extreme values  $e_1$  and  $e_2$ ”. Examples of initial selections corresponding to ALL and ANY operators are depicted in Fig. 3.6 and Fig. 3.7 respectively. The brush values for both examples are nodes 3 and 7. The selected nodes are highlighted by a shaded region. In the second phase, the initial selection is propagated up towards the

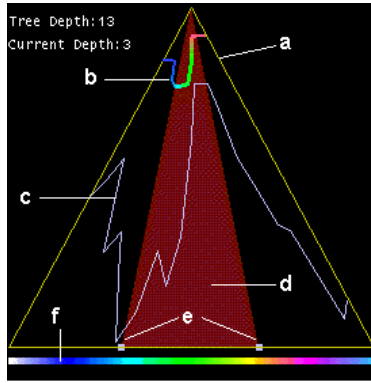


Figure 3.5: Structure-based brushing interface in XmdvTool. (a) Hierarchical tree frame; (b) Contour corresponding to current level-of-detail; (c) Leaf contour approximates shape of hierarchical tree; (d) Structure-based brush; (e) Interactive brush handles; (f) Color map legend for level-of-detail contour.

root based on what we termed an *ALL* semantic: “select nodes that have *all* their children already selected” (other semantics such as *ANY* or *MOST* are also possible [40]). The second computation phase, the vertical selection, consists of refining the set of nodes generated in phase one. Basically, the nodes on the desired level-of-detail are only retrieved out of the whole phase one selection. A formal definition of structure-based brushes can be found in Appendix A.

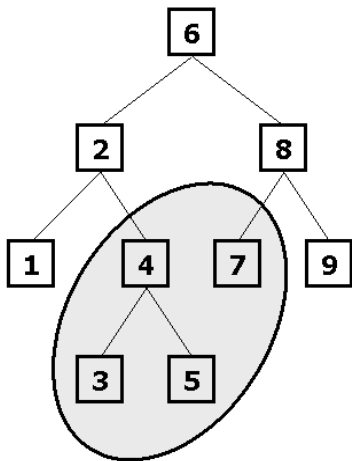


Figure 3.6: ALL initial selection with brush values 3 and 7.

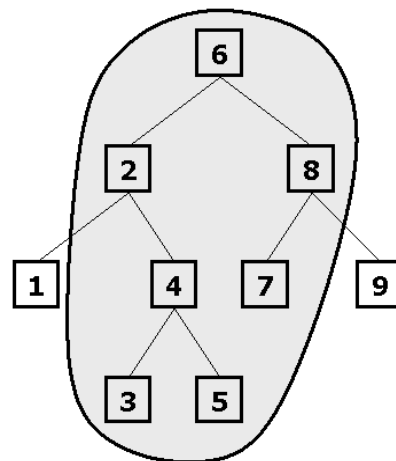


Figure 3.7: ANY initial selection with brush values 3 and 7.

The brush operations, as described above, are inherently recursive. Recursive processing in relational database systems can be extremely time consuming and thus unsuitable for interactive applications. In Chapter 4 we develop equivalent but non-recursive computation methods for setting structure-based brushes based on assigning some pre-computed values to the nodes that recast retrievals as range queries.

### 3.3.2 Geometric Representation

Based on structure-based brush definition, we can extend partition maps to incorporate information about the level value also. The objects get now a spatial representation. An example of a four level hierarchy is presented in Fig. 3.8. We called this type of representation a *2-D hierarchy map*. Hierarchy maps are especially useful when we generalize the concept of level of detail to extend to any type of monotonic function. Such an example is presented in Fig. 3.9. In this case, two level values (initial and final) need to be stored for each object. The semantics of the structure-based brushes changes under hierarchy maps. As we will see in Chapter 4, it reduces to a containment test on both dimensions. A typical example would be “select all points that touch the level of detail L and the brush interval  $(x, y)$ ”.

### 3.3.3 Multidimensional Extension

The structure-based brushes, as just introduced, imply that a natural order of the base data points (leaf nodes) exists. This may not always be the case. In fact, the space upon the initial selection is performed is one dimensional. Principally, the initial selection can be performed in an arbitrary  $k$ -dimensional space, if that is preferable for the user. Particularly, we anticipate that an  $n$ -dimensional selection will be useful. Moreover, 2-D hierarchy maps naturally generalize to  $n$ -D hierarchy maps by using  $n$ -dimensional

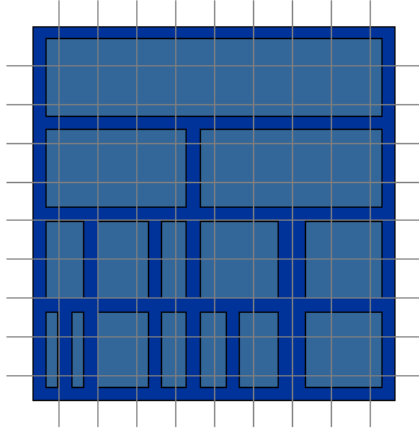


Figure 3.8: 2-D hierarchy map. Uniform levels of detail.

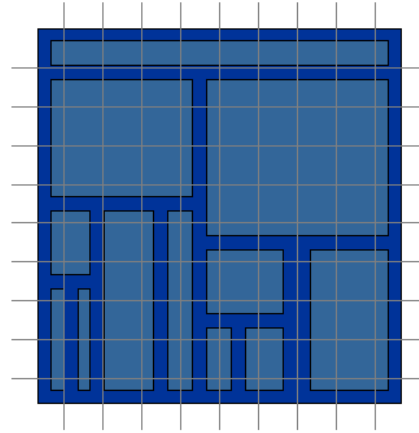


Figure 3.9: 2-D hierarchy map. Arbitrary level of detail function.

objects instead of rectangles. This extension, however, is not implemented in the current version of XmdvTool.

### 3.4 Model Abstraction

We now introduce a formal model characterizing the salient features of the proposed techniques, thus establishing the applicability and scope of our solution. The input space is composed of entries in the extent ( $x$ ) dimension and in the level ( $y$ ) dimension. Since the two dimensions are independent, the space is actually a Cartesian product of entries. We can envision this space by overlapping a 2-D grid over the tree hierarchy (Fig. 3.10). In this representation, a selection is a sequence of consecutive regions with the same level value (Fig. 3.11). The data points are *spatial* objects whose distribution may be unknown and which can be retrieved by a so called *query mechanism*. Thus, there must be a containment criterion that specifies for each object whether it is included in the current selection window or not. In order for us to be able to implement a structure-based brush as previously specified, an order is enforced on the set of leaf nodes and in general



on the nodes with the same level-of-detail value.

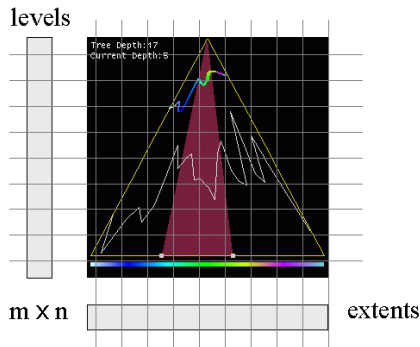


Figure 3.10: Selection space abstraction.

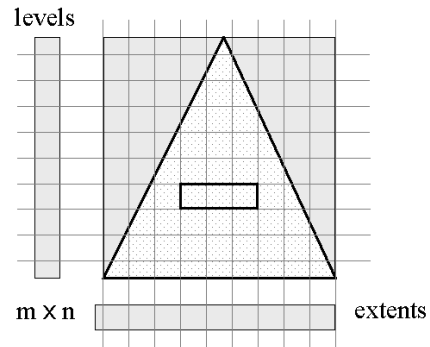


Figure 3.11: Active window in the selection space.

An example of transforming a tree structure into a *navigation space* (i.e., the set of spatial objects) is given below. The tree (Fig. 3.12) is first represented as a 2-D hierarchy map and then overlapped on a 2-D grid of integers.

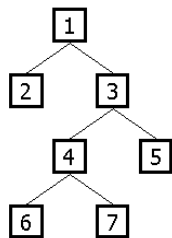


Figure 3.12: A tree example.

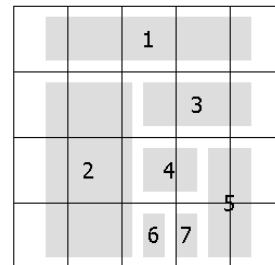


Figure 3.13: Navigation on a tree support set.

The set of characteristics below identifies the requirements that a system needs in order for our approach to apply (besides providing the naming conventions):

1. *Navigation* consists of continuously changing a selection window (called the *active window*) defined over an  $n \times m$  grid of integers (called the *navigation grid*).

2. On both axes of the *navigation grid*, the intervals are indexed rather than the points. Thus, on the  $x$  axis (called the extent axis) the intervals are denoted as  $e_i$ , while on the  $y$  axis (called the level axis) the intervals are denoted as  $L_k$ . The grid is therefore a set of rectangle regions of the form  $(e_i, L_k)$  (Fig. 3.14).

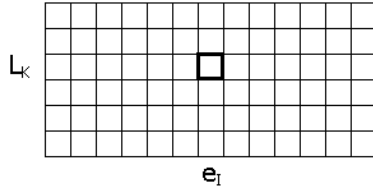


Figure 3.14: Navigation grid.

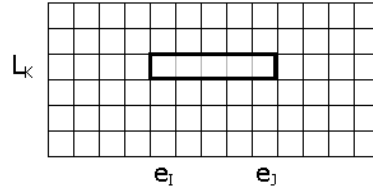


Figure 3.15: Active window.

3. An *active window* is a “compact” selection of points  $\{(e_i, L_k), (e_{i+1}, L_k), \dots, (e_j, L_k)\}$  on the same level (Fig. 3.15). Thus, an active set is specified by a triplet of the form  $(e_i, e_j, L_k)$ .
4. Each *active window* uniquely identifies a set of spatial objects (called the *active set*) being selected among the objects of a given set (called the *base set* (Fig. 3.16)).

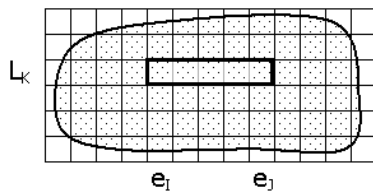


Figure 3.16: Base set.

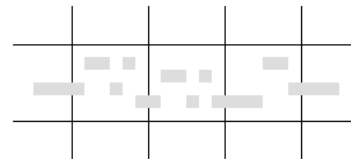


Figure 3.17: Objects on the same level are totally ordered.

The following properties about the base set are assumed:

- There is a partial order relationship defined on the objects of the base set. However, there is a total order relationship among objects on the same level (Fig. 3.17).

- Any active set can contain 0, 1 or multiple objects. This number, however, is not known in advance.
  - An object can spread over multiple regions on the same level and can belong to multiple levels. Thus, the active windows are not additive, i.e., the union of two active sets corresponding to two windows  $W1$  and  $W2$  is not necessarily the same as (although included in) the active set corresponding to  $W1 \cup W2$ . Moreover, the active sets for two disjoint active windows are not necessarily disjoint.
5. The *active window* may only change incrementally, i.e., only one of the three parameters can change at a time and only by a single unit. This is an essential property exploited by our memory management strategy, as shown in Section 5.

# Chapter 4

## Query Specification and Processing

The question addressed in this section is “how do we translate the visual exploration operations into database operations”. For this purpose we have developed a technique called the *MinMax tree* [40]. The method places the recursive processing in a *precomputation* stage, when labels are assigned to all nodes. The labels provide a containment criterion: simply by looking at the parameters of an active window and at a node’s label, we are able to determine whether that node belongs to the active selection or not.

### 4.1 MinMax Hierarchy Encoding

A MinMax tree is a  $n$ -ary tree in which nodes correspond to open intervals defined over a totally ordered set, called an *initial set*. The leaf nodes in such a tree form a sequence of non-overlapping intervals. The interior nodes are unions of intervals corresponding to their children.

The initial set can be continuous (such as an interval of real numbers) or discrete (such as a sequence of integers). In either case, the nodes are labeled as pairs of values: the extents of their interval. As the intervals are unions of child intervals, it follows that a node

will be labeled with the minimum extent of its first interval and the maximum extent of its last interval, i.e., a node  $n$  having for example two children  $c_1 = (\alpha, \beta)$  and  $c_2 = (\gamma, \delta)$  will be labeled as  $n = (\alpha, \delta)$  (Appendix B). Hence, the trees are called MinMax. Examples of MinMax trees are depicted for a continuous initial set in Fig. 4.1 and for a discrete initial set in Fig. 4.2.

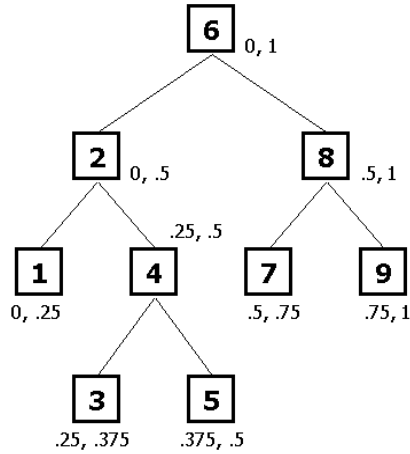


Figure 4.1: A continuous MinMax tree.

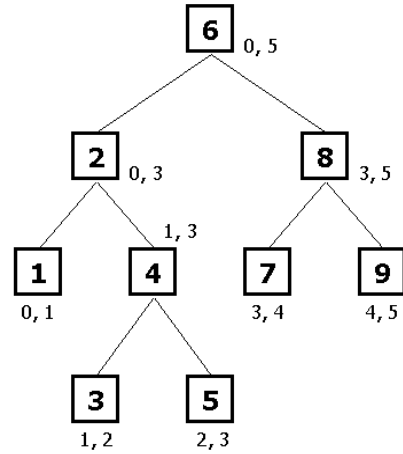


Figure 4.2: A discrete MinMax tree.

Essentially, the process of labeling the nodes is a recursive one. The intervals are computed and assigned off-line at the time the hierarchy is created and their value and distribution (as well as the tree structure itself) depend on the clustering method. Specifically, the interval size and the distribution are influenced by whether the hierarchy is created bottom-up or top-down. In the bottom-up case, the leaf intervals have the same size, while in the top-down case, the node intervals on the same level have the same size. Fig. 4.1 and Fig. 4.2 present an example of a top-down tree and an example of a bottom-up tree respectively.

An important property of a MinMax tree is captured in Theorem 1 and will be further exploited when implementing the navigation operations.

**Theorem 1** *Given a MinMax tree  $T$  and two nodes  $x$  and  $y$  of  $T$  whose extent values are  $(x_1, x_2)$  and  $(y_1, y_2)$  respectively, node  $x$  is an ancestor of node  $y$  if and only if  $x_1 \leq y_1$  and  $y_2 \leq x_2$ .*

The theorem is based on the intuition that each node in the tree is included in its parent as an interval (as constructed). A proof of the theorem is given in Appendix B.

## 4.2 Query Processing Using MinMax

Data is represented as a relational table HIER. According to the the previous section, HIER incorporates  $L$  (the node level),  $X$  (the minimum extent) and  $Y$  (the maximum extent) as well as  $n$  aggregate values:

$$\text{HIER}(L, X, Y, a_1, \dots, a_n)$$

### 4.2.1 Static Tree Hierarchies

In this section we give an implementation of the navigation operations in the case of a static tree hierarchy, i.e., no updates are present during navigation. First, we notice that any tree can be labeled as a MinMax tree if, for example, we start with an arbitrary continuous initial interval as the root and recursively divide it into equal sub-intervals, each sub-interval being assigned to a child (see, for example, the binary tree in Fig. 4.1).

#### **ALL Structure-Based Brushes**

Having the hierarchy labeled as a MinMax tree, we can implement an ALL structure-based brush (as introduced in Section 3) as a non-recursive operation based on the following property.

**Theorem 2** Given the brush values  $v_{min}$  and  $v_{max}$ , an ALL structure-based brush generates the union of all nodes  $n = (n_1, n_2)$  whose extents are fully contained in the brush interval  $(v_{min}, v_{max})$ , i.e.,  $(n_1, n_2) \subseteq (v_{min}, v_{max})$ .

The selection defined by an ALL structure-based brush for the example in Fig. 4.1 and the brush values 3 and 7 is visually depicted in Fig. 4.3. The selected nodes in the figure are underlined. A proof of Theorem 2 is given in Appendix B.

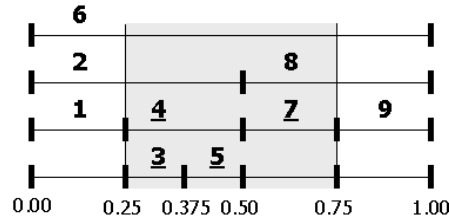


Figure 4.3: An ALL structure-based brush.

The ALL structure-based brush for a hierarchy labeled as a MinMax tree is now a simple range query, expressed in SQL2 as:

```
select  *
from    hier
where   X >= :v_min
and     Y <= :v_max
and     L = :level;
```

### ANY Structure-Based Brushes

An ANY structure-based brush as defined in Section 3 can also be implemented as a non-recursive operation. The non-recursive computation method is based on Theorem 3.

**Theorem 3** Given the brush values  $v_{min}$  and  $v_{max}$ , an ANY structure-based brush generates all the nodes  $n = (n_1, n_2)$  whose intersection with the brush interval  $(v_{min}, v_{max})$  is not empty, i.e.,  $(n_1, n_2) \cap (v_{min}, v_{max}) \neq \emptyset$ .

The property states that all nodes that “touch” the brush interval are selected. As shown in Fig. 4.4, this is intuitively true, all the underlined nodes (that are “touched”

by the shaded brush area) are part of the ANY structure-based brush as presented in the example in Section 3. A proof of Theorem 3 is given in Appendix B.

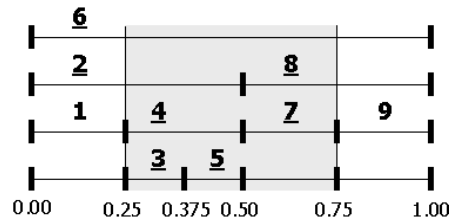


Figure 4.4: An ANY structure-based brush.

The non-recursive query for an ANY structure-based brush defined over a MinMax tree is therefore of the form:

```
select  *
from    hier
where   X < :v_max
and     Y > :v_min
and     L = :level;
```

Clearly, the above technique is powerful when the tree structure remains unchanged during exploration. However, in practice nodes often need to be added or removed dynamically. The next subsection addresses the case of a dynamic hierarchy.

## 4.2.2 Dynamic Tree Hierarchies

In a dynamic hierarchy, the tree (graph) structure changes during exploration. The type of updates we consider in this section are adding new nodes (as leaves) and deleting existing nodes. If the node to be deleted is an inner node, then we interpret this to mean that the whole sub-tree rooted at that node is removed.

Deleting nodes (sub-trees) in a MinMax tree does not require special computation (such as rearranging the trees or re-labeling the nodes) in order to preserve the properties of the MinMax trees. Deleting a subtree rooted at the node  $n = (n_1, n_2)$ , for example, is similar to setting an ALL structure based brush with the brush values  $n_1$  and  $n_2$ :



```
delete from hier
where X >= :v_min
and Y <= :v_max;
```

When inserting a new node  $n$ , however, the out-degree of the parent node changes and all siblings of  $n$  (and their descendents) need to update their intervals. We say that the node interval “splits”. In order to increase the efficiency of this process, we use a two-step method. First, we delay the interval splitting by inserting some “gaps” in the tree nodes (Section 4.2.2). Second, we re-label the affected nodes when splitting by using a fast non-recursive method (Section 4.2.2).

### **De-Compacting The Tree**

Let us consider the case of a node  $n = (n_1, n_2)$  that has three children. The method so far divides the  $(n_1, n_2)$  interval into three sub-intervals. If a fourth node is inserted,  $n$  has to split. If, instead of 3, we first had divided  $n$  into more, let’s say 5, intervals, the fourth node could have been added without any problem, and thus the splitting would have been delayed.

Based on this idea, we chose the allocation management suggested in [10]. We first label the MinMax tree as an N-ary tree (we say that we “allocate” N positions for each node). Then, when a new node  $k + 1$  is inserted in a node  $n$  which has only  $k$  positions allocated,  $n$  just doubles its interval (it expands from  $k$  to  $2k$  positions) (Fig. 4.5). By using an amortized analysis, this allocation strategy was proven to be optimal when the maximum number of elements that has to be stored is unknown (and cannot be estimated) ([10]).

### **Re-Labeling The Nodes**

When a node  $n = (n_1, n_2)$  splits, a re-labeling process takes place. The extents of all nodes in the sub-tree rooted at  $n$  have to be recomputed. But, the sub-tree can be selected based

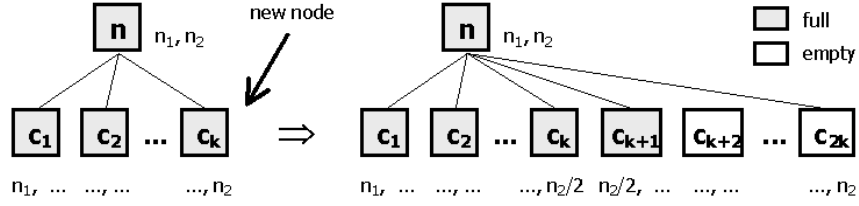


Figure 4.5: The allocation strategy.

on the  $n_1$  and  $n_2$  values. Moreover, for the selected tuples, an affine transformation can be utilized to update the extent values:

```

update hier
set      X = :v_min+(X-:v_min)/2
         Y = :v_max+(Y-:v_max)/2
where   X >= :v_min
and     Y <= :v_max;

```

### 4.2.3 Arbitrary Hierarchies

An arbitrary hierarchy is one in which a node can have more than one parent, i.e., a non-tree acyclic di-graph (Fig. 4.6). One example application of arbitrary hierarchies is CAD/CAM part hierarchies. In these applications our structure-based brushes have an interesting semantic. Given a set  $S$  of basic components (the leaf nodes), an ALL structure-based brush defines the set of super-components that can be manufactured using only parts from  $S$ . An ANY structure-based brush gives the super-components that need to use any (at least one) part from  $S$ .

One extension of our method can be designed to handle arbitrary hierarchies, too. In an arbitrary hierarchy, more than one interval can be assigned to a node. For example, by using a discrete bottom-up labeling for the tree in Fig. 4.6, two intervals are assigned to node 5, as shown in Fig. 4.7.

This case is handled by inserting two copies of node 5 into the HIER table. Thus, the first copy will be assigned the first interval and labeled (0, 1) while the second copy will

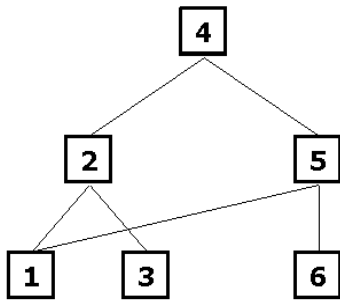


Figure 4.6: An arbitrary hierarchy.

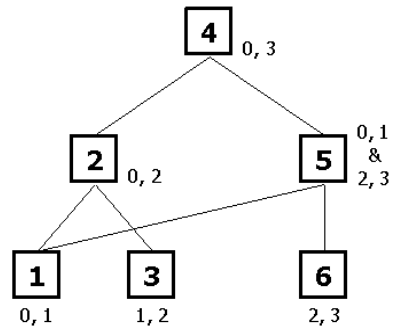


Figure 4.7: Bottom-up labeling of an arbitrary hierarchy.

be assigned the second interval and labeled (2, 3). It is important to notice that the number of additional tuples to be inserted in the HIER table depends on the ordering of the nodes. For example, if nodes 1 and 3 change their position then node 5 will be labeled (1,3) and thus no duplicate copies need to be inserted. However, in this paper we do not address the problem of how to organize the hierarchy nodes in order to decrease the number of stored tuples.

If more than one copy of the same node exists in the hierarchy, the (non-recursive) implementation queries for the structure-based brushes change. Thus, because the same nodes may occur multiple times in the table having different interval values, some of them possibly inside and some of them possibly outside the brush interval, the ALL brush becomes “select the nodes that do not have intervals outside the brush interval”:

```
select distinct *
from hier
where L = :level
except
select *
from hier
where X > :v_max
or Y < :v_min;
```

The ANY structure-based brush query also changes to handle duplicates:

```
select distinct *
from hier
where X < :v_max
and Y > :v_min
and L = :level;
```

While still non-recursive, the new queries are significantly more expensive than those designed for tree hierarchies. Therefore, when no duplicate copies are used, the queries designed for tree hierarchies are preferred.

# Chapter 5

## Memory Management

### 5.1 Caching

The questions addressed in this section are “how do we organize the local memory?” and “when and what data do we request from the database?”. The memory organization is critical in interactive applications since it influences the performance of the subsequent operations. When a request for new objects is issued by the front-end, the difference between the new active set (i.e., the set of objects just selected) and the current content of the buffer has to be quickly computed. Thus, we need to be able to know in each moment what data resides in the memory without fully traversing the buffer.

A significant difference in the buffer management is made by whether the buffer is large enough to store all the objects in the active set or not. We refer to these two cases as database intensive (DBI) and database semi-intensive (DBSI). We are primarily concern about the DBSI case when the active set of objects does not occupy the whole space available, although we also propose a technique that would handle the DBI case.

When there is still space available in the memory and the system is idle, we can load additional data from the slow memory (disk). If that data, in full or partially, is needed

further (before it gets replaced) then the time that would have been spent bringing it into the buffer is a gain in the system’s overall latency.

For this purpose, we designed and implemented a speculative, adaptive, and non-pure strategy for prefetching. The prefetcher is speculative in that it doesn’t use any *explicit* information about the next operations but tries to guess them. Adaptive refers to the ability to change the prefetching strategy dynamically, as more information is available in the system. In our case, as shown in Chapter 6, we do not fully implement the adaptability part. However, more than one strategy has been proposed, and as we will show later, strategies perform better when more information is available. The prefetcher is non-pure in that it implements a non-penalty policy, in which user actions preempt the prefetching decisions.

### 5.1.1 Semantic Caching

Semantic caching is a high level type of cache in which queries are cached rather than pages or tuples. A characteristic of the objects that are placed in the buffer is that they are not referenced by their IDs when accessed by the front-end. In other words, the front-end doesn’t ask for objects using requests such as  $ID = x$  or  $ID = y$ ; instead, it passes a query  $q_{requested}$  to the back-end: “are the objects with these characteristics (within this brush) available?”. Thus, although there are object attributes (the extents) that uniquely identify each entry, a classical lookup for a cache key is not possible when testing whether an object is in the buffer or not. Instead, a set of queries  $q_{content}^i$  is associated with the buffer, similar to *semantic caching* [12]. The query  $q_{requested}$  is then compared with each  $q_{content}^i$  to determine what objects from  $q_{requested}$  are not in  $q_{content}^i$ , and those objects are retrieved next. This difference results in new queries ( $q_{+}^i$ ) that correspond to those *to be loaded next* objects.

The problem of determine the  $q_{+}^i$  queries is usually known as *query folding* [31]. It has

been shown that the problem is reducible to the query containment problem [4]. Query containment is undecidable in the general case but decidable in the case of conjunctive queries [6]. As shown in Chapter 4 the queries in our case are all range queries, and therefore conjunctive.

Special attention has to be paid in a semantic caching environment to not allow duplicates in the buffer. Thus, when more than one  $q_{content}^i$  query is stored in the buffer, they are *forced* to be disjoint. This means that a new  $q_{requested}$  query will modify the semantic of the existing  $q_{content}^i$  queries such that they do not refer to any common objects any more. As we will show later, we partition our objects based on the level value. This partitioning makes the task of testing for containment reduce to checking the extent values only.

In order to make the object additions and subtractions efficient, we store the objects in the buffer ordered by their extent value. The order can be ensured by the *query mechanism* itself or can be added as a new processing step. In our case we can request that the objects in all queries (as defined in Section 4) be retrieved in order by adding an *ORDERED BY* clause to or MinMax-derived SQL queries. This SQL clause will not require extra processing time if we store the objects in the database ordered by their extent values (left, for example). It would require only minimal extra processing if we store the objects unordered but have an index built.

A problem that all cache strategies need to solve is the cache replacement policy, i.e., to determine what objects have to be removed from the cache to make room for new objects. The first step in implementing a replacement policy is to provide an estimation strategy able to measure the likelihood that an object will be needed in the near future. The estimation strategy, also called a *predictor*, is usually based on heuristics, probabilistic models, or some recorded statistics. In our case we use a probability function. The probability function also defines a partition on the set of objects.

The objects in the memory are thus partitioned based on both the level and the proba-

bility value. An efficient way to implement this is to use two hash (bucket) tables and hash (distribute) the objects into the appropriate buckets. The objects in the same bucket are connected by a double linked list. We will explain the functionality of this organization in Section 5.1.3.

## 5.1.2 Probabilistic Model

Let's consider a navigation grid  $\Delta = (1..I) \times (1..K)$  as introduced in Chapter 3. Each point from the support set, and thus each region  $(e_i, L_k)$  from the navigation grid, has an associated probability  $\mathcal{P}(m, i, k)$  that measures the likelihood that the point will belong to the active set after user's next  $m$  operations. Also, a probability  $\mathcal{P}^*(m, i, k)$  will measure the likelihood that the point will belong to the active set at any time during the next  $m$  operations. Obviously, we have:  $\mathcal{P}^*(m, i, k) = \bigoplus_{t=0}^m \mathcal{P}(t, i, k)$ , where  $\oplus$  is a probability sum, i.e.,  $p_1 \oplus p_2 = p_1 + p_2 - p_1 p_2$  (from the principle of inclusion and exclusion).

The *lookahead parameter* (LA) is the number of operations considered in advance when computing the probabilities  $\mathcal{P}$  and  $\mathcal{P}^*$ , i.e., the parameter  $m$  from the definitions above.

We say that the *monotonicity property* (MP) holds on level  $k$  for a function (distribution)  $f$  if there exists an extent value  $E = E(k)$  such that  $f(\cdot, k)$  is monotonically increasing for values less than  $E$  and monotonically decreasing for values greater than  $E$ , i.e, for each  $j_1 < j_2 \leq E$  we have  $f(j_1, k) \leq f(j_2, k)$  and for each  $j_1 > j_2 \geq E$  we have  $f(j_1, k) \geq f(j_2, k)$ .

The LA parameter dictates how many operations the predictor will predict. In general, the bigger LA is the more speculative the system becomes and thus the more errors are involved. We used in our implementation an LA equal to 1. If the prediction model is very accurate (generates high confident predictions) an LA equal to 2 may eventually be used. We don't anticipate though that a value greater than 2 will ever be used.



We say that probabilities assigned to the objects are *operation-driven* if they are based on the probabilities that the predictor assigns to the possible next operations. In our case we have six possible operations (restricting or enlarging any of the three active window's parameters). Let us assume, for example, that we have an active window  $\omega = (i_1, i_2, k)$  and from this configuration, going left with  $i_1$  is 50% probable, going up with  $k$  is 25% probable, and so on. Then, objects in  $(i_1, i_2, k)$  will have a probability of 1, objects in  $(i_1 - 1, i_1, k)$  will have a probability of .50, objects in  $(i_1, i_2, k - 1)$  a probability of .25, and so on.

**Theorem 4** Let  $\Delta = (1..I) \times (1..K)$  be a navigation grid. For any operation-driven probability model and any lookahead  $LA = 0, 1$  or  $2$ , the MP holds on each level  $(1..K)$  for both  $\mathcal{P}$  and  $\mathcal{P}^*$ .

PROOF: (1) We consider  $\mathcal{P}$  first. Let the six operations ( $k \uparrow, k \downarrow, i_1 \leftarrow, i_1 \rightarrow, i_2 \leftarrow, i_2 \rightarrow$ ) be numbered 1 to 6 (in order).

- LA=0. For any given selection there are two regions of equal probability (Fig. 5.1).  
MP holds trivially in this case.

- Region 0:  $\mathcal{P} = 0$  (unselected points)
- Region 1:  $\mathcal{P} = 1$  (selected points)

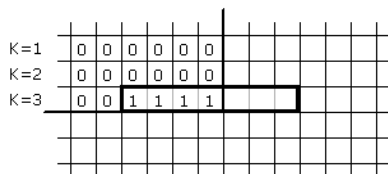


Figure 5.1: LA=0: two regions of equal probability, 0 and 1.

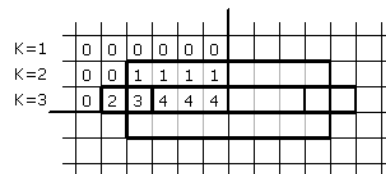


Figure 5.2: LA=1: five regions of equal probability, 0, 1, ..., 4.

- LA=1. The six operations that possibly change a given selection define five regions of equal probability (Fig 5.2). Let  $p_1, \dots, p_6$  be the probabilities associated with the operations. Then,  $\mathcal{P}$  can be computed for each region.

- Region 0:  $\mathcal{P} = 0$
- Region 1:  $\mathcal{P} = p_1$
- Region 2:  $\mathcal{P} = p_3$
- Region 3:  $\mathcal{P} = p_3 + p_4 + p_5$
- Region 4:  $\mathcal{P} = p_3 + p_4 + p_5 + p_6$

On levels, we have three possible types of configurations (distributions); MP holds on all of them.

- Level 1:  $0, 0, \dots, 0$
- Level 2:  $0, 0, p_1, \dots, p_1, 0, 0$
- Level 3:  $0, p_3, p_3 + p_4 + p_5, p_3 + p_4 + p_5 + p_6, \dots, p_3 + p_4 + p_5 + p_6, p_3 + p_4 + p_5, p_5, 0$

- LA=2. Let  $a_1, \dots, a_6$  be the probabilities associated with the first user operation and  $b_1, \dots, b_6$  the probabilities associated with the second. Similarly, there are 10 regions of equal probability (Fig. 5.3). For all these regions  $\mathcal{P}$  is computed below. It is again easy to see that the MP holds in this case also.

- Region 0:  $\mathcal{P} = 0$
- Region 1:  $\mathcal{P} = a_1 b_1$
- Region 2:  $\mathcal{P} = a_1 b_3 + a_3 b_1$
- Region 3:  $\mathcal{P} = a_1 (b_3 + b_5 + b_6) + (a_3 + a_5 + a_6) b_1$
- Region 4:  $\mathcal{P} = a_1 (b_3 + b_4 + b_5 + b_6) + (a_3 + a_4 + a_5 + a_6) b_1$
- Region 5:  $\mathcal{P} = a_3 b_3$
- Region 6:  $\mathcal{P} = a_3 (b_5 + b_6) + (a_5 + a_6) b_3$

- Region 7:  $\mathcal{P} = a_1b_2 + a_2b_1 + a_4b_3 + (a_3 + a_5 + a_6)(b_3 + b_4 + b_5 + b_6)$
- Region 8:  $\mathcal{P} = a_1b_2 + a_2b_1 + a_4(b_3 + b_5 + b_6) + (a_3 + a_5 + a_6)(b_3 + b_4 + b_5 + b_6)$
- Region 9:  $\mathcal{P} = a_1b_2 + a_2b_1 + (a_3 + a_4 + a_5 + a_6)(b_3 + b_4 + b_5 + b_6)$

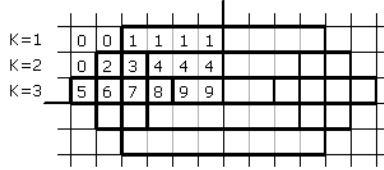


Figure 5.3: LA=2: ten regions of equal probability, 0, 1, ..., 9.

(2) Let us consider  $\mathcal{P}^*$  now. For a given selection, there exists at least one extent value  $E$  (the median extent of the active set) that doesn't depend on the level or the lookahead value ( $\leq 2$ ) and that makes the MP hold for  $\mathcal{P}$  on all levels. Therefore for each  $k$ ,  $\mathcal{P}(\cdot, k)$  is monotonic on both  $(-\infty, E)$  and  $(E, +\infty)$  for any lookahead value ( $\leq 2$ ). Since  $\mathcal{P}^*(\cdot, k)$  is a probability sum ( $\oplus$ ) of monotonically increasing functions on  $(-\infty, E)$  and monotonically decreasing functions on  $(E, +\infty)$ , it necessarily ( $p_1 \oplus p_2 \geq \max(p_1, p_2)$ ) has the same monotonicity property and therefore the MP holds on level  $k$  (q.e.d.)

In what follows, a probabilistic model with a lookahead value of 0, 1 or 2 is assumed (so that MP holds on all levels for both  $\mathcal{P}$  and  $\mathcal{P}^*$ ).

### 5.1.3 Cache Replacement

As shown in Section 5.1.1, the buffer is first organized as a *bucket table* based on the probability values. The objects in the buffer are hashed by rounding, based on a fixed number of values (a given precision). The buckets will thus have values ranging uniformly from 0 (an open entry) to 1 (an object being currently in the active set). The objects in the

same bucket are linked by a double linked list. Independently, the buffer is also hashed based on the level value. Again, we have a bucket table with as many buckets as the level values. The objects in the same bucket are linked by a double linked list. In addition, the header keeps a pointer to the last element in the list. An invariant of the model is that, on each level, the objects are ordered (and linked in the linked list) by their extent values. This is assumed to be always possible, as discussed in Chapter 3.

In what follows we will focus on the operations that this structure needs to support. The main task of a cache replacement policy is to find in the buffer the entries that have the lowest probability of being used and to remove them when more room is needed. This operation needs to be efficient, since it occurs frequently.

When new objects are brought in they have to comply with the internal organization. Updating the hash tables is then required.

When a request is issued by the front-end, a containment test is performed. The system first checks whether the requested data reside entirely in memory or not. In case it doesn't, a *compensation* query has to be send to the *loader*, an agent that fetches the data from the persistent storage.

The front-end may also send “refresh” queries when all objects within the current selection are needed.

An important requirement of the system that comes from its interactive nature is that the user needs to be able to preempt the other agents' actions. Thus, when the current selection changes, the loading process is interrupted and will only restart after recomputing the new probability values for the objects.

In conclusion, the buffer access operations can be summarized as:

A: **Remove old objects.** Get the objects with the lowest probability that reside in the buffer (and further remove them one at a time when more room in the buffer is needed).

- B: **Bring new objects.** Place an object from the database *cursor* into the memory buffer (and rehash the buffer entry).
- C: **Display active set.** Get those objects from the buffer that form the active set (and send them to the graphical interface to have them displayed).
- D: **Recompute probabilities.** Recompute the probabilities of the objects in the buffer once the active window gets changed (to ensure accurate predictions in the future).
- E: **Test containment.** Test whether the new active set fully resides in the buffer and gets the missing objects (if any) from the support set (when a new request is issued).

In the remainder section we will show how these operations are implemented in our buffer strategy.

A speed up in the buffer processing can be achieved by using a simplified version of the probability-based bucket table. Thus, instead of storing all objects of the same probability in one bucket, we only store the ones which are extreme elements (first and last) in the level based lists. However, for a better understanding of the problem I will describe both cases.

As an example let us consider the navigation grid displayed in Fig. 5.4. We have here twelve regions of equal probability, the active window covering the middle two ones. For simplicity we consider that only one object resides in each region. We also number the objects from 1 to 12. The picture presents only three levels (1, 2, and 3). Also, probabilities are assigned to each region and implicitly to each object, based on a “operation-driven” probability model. Thus, objects 6 and 7 have a probability of 1, there is 40% chance that the window expands to the left, and so on. In this example a probability precision of 0.1 is assumed, and consequently 10 probability-based buckets are used. The probability table is reduced; one can see here for instance that only 5 and 8 are hashed out of the entire level 2.

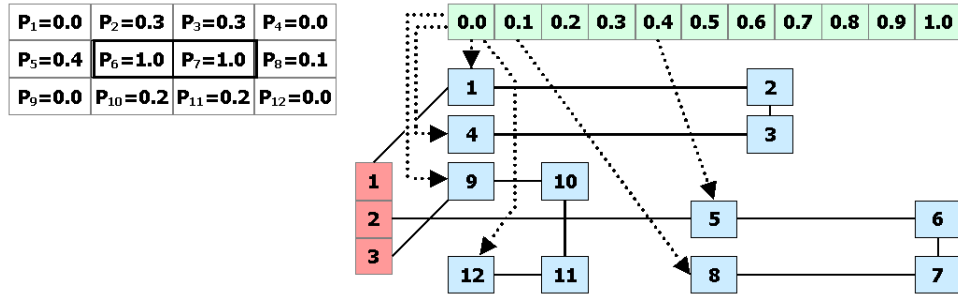


Figure 5.4: Buffer content for a three level, twelve object example in case of a reduced probability table.

An important assumption made at this point is that the query mechanism is able to provide the objects from the active set in both the increasing and the decreasing order of their extent value. If not, a sorting stage has to follow all calls to the query mechanism.

The main idea behind the buffer access strategy is to keep the sets of objects on each level always *convex* in the buffer (with respect to the relation of total order defined among the objects of the same level). This is possible due to the fact that the lowest probability objects which need to be replaced (and thus removed from that level list) are always at the extreme of the list.

In what follows, the implementation of the buffer access operations is described. For a better understanding, a full version of the probability table is assumed first. The complexity of these operations is presented in Appendix C.

Operation A – **remove old objects** is equivalent to retrieving elements from the non-empty probability buckets in increasing order of their bucket value. The operation thus requires a scan of the probability table interleaved with traversals of the bucket lists.

Operation B – **bring new objects** is equivalent to hashing an entry with respect to both its level and its probability value. Hashing an entry with respect to its level value is done in exactly one or two operations. Since the set of entries on each level is convex and contains the higher probability objects of the level, then the newly added object, which has the highest probability among the objects not yet in the buffer, will necessarily be at

the extreme. The entries on each level are ordered based on the extent value. Therefore the new object is either the new first element on that level's list if it is less than the previous first object, or the new last element if (necessarily) it is greater than the previous last element. Hashing with respect to the probability unfortunately takes  $m/2$  operations in the average case, where  $m$  is the length of the current probability list. We need to make sure that the object is correctly inserted (with respect to its extent value) in the sequence of elements having the same level in its probability bucket, so that removing the elements one by one from the probability list leaves all the affected level lists still convex. The efficiency will be improved though by using a reduced probability list.

Operation C – **display active set** is simply read all entries from the probability 1 bucket (last bucket). The number of buffer accesses is the number of elements in the bucket.

Operation D – **recompute probabilities** requires at least one complete scan of the data. The objects preserve their level value so no change of the level lists is needed. However, the probability table needs to be rebuilt, and this takes  $n + n^2/2$  operations.  $n$  is for deleting the lists (this can be done together with the probability recomputation step) and  $n^2/2$  for creating the new ones (it is basically one list insertion for each object).

Operation E – **test containment** is composed of two steps. The inclusion test is ensured by the convexity property. An active set corresponding to an active window  $(e_1, e_2, k)$  is included in the buffer, if and only if the list corresponding to level  $L_k$  is  $o_{k1}, \dots, o_{kn}$  and the left extent (geometrical extremity) of  $o_{k1}$ ,  $left(o_{k1})$  is less than or equal to  $e_1$  and the right extent of  $o_{kn}$ ,  $right(o_{kn})$  is greater than or equal to  $e_2$ . If not included, the difference between the intervals  $(e_1, e_2)$  and  $(left(o_{k1}), right(o_{kn}))$  gives us the next request(s) to be addressed to the *query mechanism*.

We now consider the case of a reduced (simplified) probability table. The complexity of these operations is also presented in Appendix C.

Operation A (revisited). The same scan as in the case of a full probability table is needed for retrieving the first non-empty bucket. However, after getting the first element in the probability list, the level list is followed this time (as long as the entries there have probabilities equal to the current bucket's probability  $p$ ). The first element with probability greater than  $p$  is inserted in a probability list (since it will remain the extreme element) and the process continues with the next element in the  $p$  probability list.

Operation B (revisited). Rehashing based on the level value takes the same number of operations as the case of a reduced probability table. Rehashing with respect to the probability however takes exactly two operations in this case. The new object will be inserted as the first element in the probability list corresponding to its probability value and the second element (or the second last) in the level list (the previous extreme) is removed from its probability list.

Operation C (revisited). Operation C does not change.

Operation D (revisited). Clearly, one complete scan is necessary for recomputing the probabilities (and this scan can also be used to delete the probability lists). Since the probability lists only contain the first and the last element in the level lists, rebuilding the probability table only takes  $2m$  operations, where  $m$  is the length of the level bucket table  $L$ .

Operation E (revisited). Operation E does not change.

#### **5.1.4 Database Intensive Case**

In this case only one level is used at a time. The level table becomes a circular list, while the probability table is not at all used since all entries have now the same probability. The only problem that needs to be addressed in this context is the synchronization between a “producer” process (the loader that fetches the data) and a “consumer” process (the front-end that displays the data). The synchronization becomes essential in this case since the



two tasks cannot be completely “serialized” any more.

In the DBI case, the number of objects in one active set at a time may be greater than the number  $n$  of entries available in the buffer. Thus, while the producer process advances in the circular list, it may attempt to overwrite some of the old objects (Fig. 5.5). This is acceptable if the objects to be overwritten have already been read by the front-end. However, to assure this functionality the model must change under the DBI assumption. Intuitively, if for instance the data in the buffer needs to be read a second time (as is the case in a *refresh* process) then the consumer changes the traversal direction. The producer is positioned at the first non-overwritten entry and also changes its traversal direction (Fig. 5.6). In this *two-way traversal* procedure we keep two pointers  $i_0$  and  $i_1$  positioned at the first and last element, respectively, in the current active set (if those elements exist in the buffer). These pointers do not however take part in the synchronization process. The synchronization simply requires that the consumer and the producer do not cross one another. Since the traversal direction is irrelevant, we implement this requirement by not allowing the two pointers to take on the same value, no matter which one follows the other.

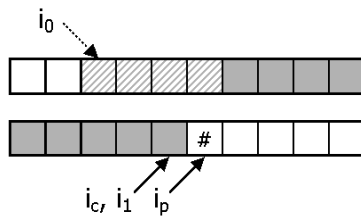


Figure 5.5: Finishing reading. Current objects are painted solid; the overwritten objects striped. Dashed lines are undefined pointers.

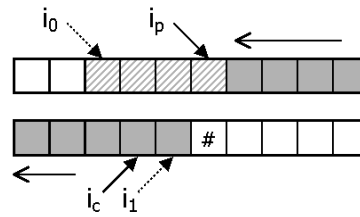


Figure 5.6: Starting re-reading. The traversal direction changes for both consumer and producer. Buffer in an inconsistent state.

A *consistent* state is one in which the buffer contains a sequence of objects that either starts with the first element of the active set (referred to a  $\alpha$ -state) or ends with the last

one (referred to as a  $\beta$ -state), i.e.,  $i_0$  and  $i_1$  are not both undefined (NULL) at the same time. An  $\alpha$ -state usually occurs after a backward traversal, while a  $\beta$ -state occurs after a forward traversal. There are six types of access operations in the DBI case. We will show that all these operations leave the buffer in a consistent state.

Refreshing (re-reading the buffer). (a) A forward traversal starts.  $i_c$  is set to  $i_0$ ,  $i_p$  to the end of the sequence (always one step before  $i_0$ ).  $i_1$  is undefined and  $i_0$  is also set to NULL. The loading process continues with the producer behind the consumer, until the end of the active set is reached and a termination character is placed.  $i_1$  is then redefined. The whole process ends when the reader reaches the termination character. The buffer is left in a  $\beta$ -state. (b) A backward traversal symmetric to (1a) begins. Changing the level. Changing the level will flush the current buffer no matter what the state is and load the new active set in a forward direction. A  $\beta$ -state is reached. Expanding the brush from left. (a) The process corresponds to a continuation of the loading process as if it were temporarily interrupted. The termination character is removed and the producer continues to place objects in the buffer. The reader starts reading elements from its previous position. The process ends again in an  $\alpha$ -state. A re-read operation of type (1a) usually follows. (b) Same as (1b). Expanding the brush from right. (a) Same as (1a). Similar with (3a) (symmetric). Restricting the brush from left. (a) The elements that no longer belong to the active set are removed from the buffer starting from  $i_0$ . When the first element of the new active set is reached, an (1a) re-read process begins. (b) Same as (1b). Restricting the brush from right. (a) Same as (1a). Similar with (5a) (symmetric).

In conclusion, except for the two cases (3a) and (4b), when a re-read is usually needed after a sequence of writings, the reading and writing time typically overlap (the operations can be completely parallelized).

## 5.2 Prefetching

### 5.2.1 General Characteristics

Computer memories are usually organized hierarchically. A two-level memory consists of a relatively small but fast cache (such as internal memory) and a relatively large but slow memory (such as disk storage). The data requested by an application must be in cache before computation can proceed. If requested data is not in the cache, then the application has to wait while the data is fetched from the slow memory to cache. The method of fetching data into the cache only when a specific request has occurred is referred to as *demand fetching*. In many applications, users spend a significant amount of time interpreting data and the processor and I/O system are typically idle during that period. If the computer can predict what data the user will request next, it can start fetching that data into cache (if not already there) *before* the user asks for it. Thus, when the user requests the data, it is available in cache, and the user perceives a faster response time.

In many interactive database applications, there is often sufficient time between user requests, and therefore the amount of data that can be prefetched is limited only by the cache size. This situation is referred to as *pure prefetching* and constitutes an important theoretical model in analyzing the benefit of prefetching. In practice however prefetch requests are often interrupted by user requests, resulting in less data being prefetched at a time. In such cases, called *non-pure prefetching*, issues of cache replacement also need to be considered. Basically, pure prefetchers can be converted into practical non-pure ones by combining them with cache replacement strategies.

An *informed* (off-line) algorithm can use knowledge about future activity. If the program generating requests is known a priori, prefetching decisions can be made off-line (as it is done in compiler-directed prefetching, for instance). An algorithm is *speculative* (on-line) if it makes its decisions based only on the history. Without a priori knowledge

or statistics of the user request patterns, as is the case of most of the interactive (such as hypertext [15]) applications, prefetching must be speculative (on-line). An important requirement of speculative prefetching is that the time spent on making prefetch decisions must be minimal.

An algorithm is *adaptive* if it has the ability to change its prefetching policy due to some run-time variations. As the access behaviour of a program may vary during its execution, changing when to issue prefetching requests or the amount of data to be prefetched may influence the performance of prefetching.

We designed and implemented a speculative, adaptive, non-pure strategy for prefetching. We will describe our approach in the remaining part of this chapter.

### 5.2.2 Strategies

Our approach is to generate a hierarchy of prefetching strategies, based on different prefetching hints. The assumption is that the predictor can discover the hints *gradually*. The approach implies an evolutionary behaviour: at the beginning, less information is available to the predictor and therefore the number of prefetching hints that it can discover (with a reasonable confidence) is also low. In time, more information (e.g., statistics) becomes available, and therefore more patterns (and implicitly hints) can be discovered. In all cases the prefetcher bases its strategy on the maximum amount of information it can find.

In our case, we assume that the predictor can discover two types of navigation patterns. Specifically, we assume that the predictor can detect if the user tends to use more frequently the current navigation direction instead of changing it and also it can detect if the data being analyzed has some regions of interest (so called *hot regions*) towards which the user will very likely go, sooner or later. Based on these assumptions, we designed three prefetching strategies: *random* (S1), *direction* (S2), and *focus* (S3). In experiments,

we also considered the case of not prefetching at all, case referred to as S0.

Strategy S1 (random) is based on randomly choosing the direction in which to prefetch next. This strategy is appropriate when the predictor either cannot extract prefetching hints or provides hints with a low confidence measure.

Strategy S2 (direction) implies that the direction of the next operation can be determined. Based on patterns extracted from the user's past explorations as well as on the parameters of the current navigation, the predictor could assign a higher probability to a particular direction versus another. Given the direction, this prefetching strategy (S2) says "prefetch as much data as you can in this given direction". The presupposition that the next direction can be determined is not arbitrary. It is very intuitive for instance that the user will continue to use the same manipulation tool for a while before changing to another one (each manipulation tool precisely determines one direction).

Strategy S3 (focus) uses information about the most probable next direction as well as hints about regions of high interest in the data space. The strategy will continue to prefetch data in the given direction. However, when a hot region is encountered the prefetcher stops prefetching in that direction. The reason is that it is very likely that the user will stop there or at least spend more time in that region.

In Chapter 7 we compare these three strategies. The results confirm our general assertion that the more information is available, the more efficient prefetching is. Thus, we can conclude that changing the prefetching strategy as soon as more patterns are discovered will improve the overall performance.

# Chapter 6

## Implementation

### 6.1 System Architecture

The work described so far has been implemented as an extension to XmdvTool 4.0. A high-level diagram of the modules added and the relationship between these modules and the original system is presented in Figure 6.1. XmdvTool 4.0 was coded in C++ with TCL/TK and OpenGL primitives. The new added modules are written in C with ProC\* (embedded SQL) primitives.

**Interaction with the original system.** The interaction with the original system (shown as GUI in Figure 6.1) has been reduced as much as possible. We have implemented what we called a prepare/iterate paradigm. Thus, when the user issues a new request, the front-end only has to inform the back-end about the request by calling *prepare*, and then retrieve the desired objects one at a time by repeatedly calling *iterate*.

**The query rewriter.** The user request arrives first to the rewriter. This module consults the information in the buffer, and decides whether the request can be immediately served or needs more information from the database. It then rewrites the original query as a set of *loading* requests.

**The query translator.** The translator takes a loading request as input and, based on the schema of the current table, translates it into SQL. The translated queries are then passed to the loader module.

**The loader.** The loader is the only module that directly interacts with the database. Its duties are to allocate and deallocate the Oracle *descriptors*, copy and format data from the Oracle structures into the application buffer, and so on. To determine which elements need to be removed from cache to make room for the new entries, the loader cooperates with the estimator.

**The estimator.** The estimator is the module that predicts the future actions of the user and assigns probabilities to the buffer entries based on those predictions.

**The prefetcher.** Running as a separate process, the prefetcher is the module that generates off-line loading requests (called prefetching requests). The prefetcher is started and stopped any time a new user request arrives. The synchronization between it and the parent process is described in the next section. The prefetcher cooperates with the estimator that provides its hints and with the rewriter that processes the prefetching requests further.

## 6.2 Threads and Synchronization

As shown in Figure 6.2, there are two threads in the system: the *main thread* and the *prefetcher*. In addition, TK comes with a complicated system of priority queues to ensure *some* fairness and optimization in the system. More precisely, TK has idle-job queues that keep track of all processes that need to be performed when the system is idle. Refreshing its *widgets* is such a process, for instance. These queues introduce a hierarchy of priorities and ensure for example that the immediate user requests preempt other processes (most of the idle-jobs are however not interruptible). Moreover, TK is able to cancel some of the requests in the queues if it can determine that those requests are not needed any more.

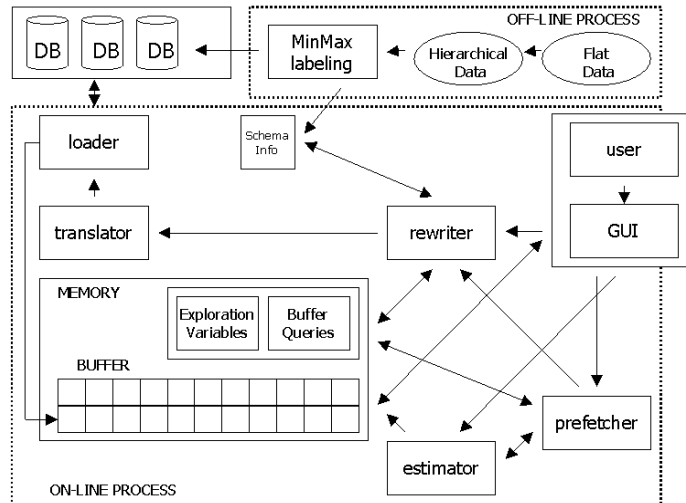


Figure 6.1: System architecture. The ovals depict the flat-file data. Dashed-line rectangles show the separation between the on-line and the off-line computation. Solid-line rectangles represent the agents. Squares represent meta-knowledge. Solid arrows show the control flow.

The priority queues in TK are important here because they explain how the GUI functions as a separate process. However, they also affect the performance, as shown in Chapter 7.

As said before, the prefetcher thread is created by the back-end when the system is idle and gets cancelled as soon as an explicit request is received. A synchronization diagram is presented in Figure 6.2. The synchronization was implemented using the *pthread* library.

### 6.3 Interacting with the Database

In order to load data from the database into the main memory we used dynamic SQL statements, specifically, dynamic SQL of “type 4”. Unlike the other types that Oracle provides, type 4 can build dynamic SQL statements that contain an unknown number of select-list items or placeholders for bind variables and can take explicit control over data-type conversion between Oracle and C types. To process this kind of dynamic SQL statements, the program had to explicitly declare *select* and *bind* descriptors (SQLDAs).



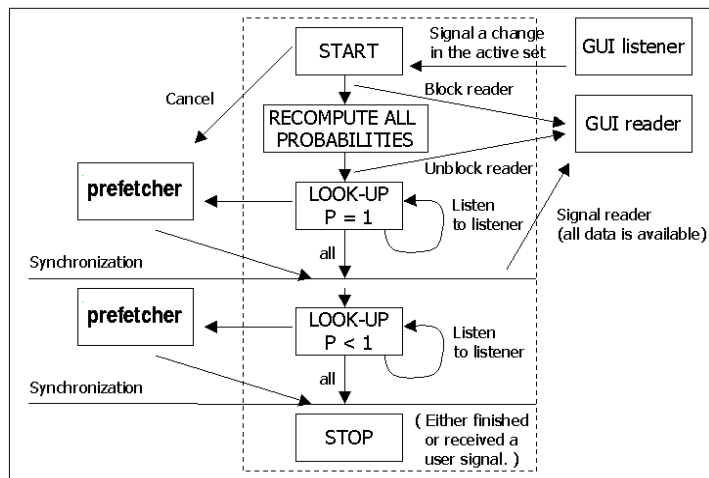


Figure 6.2: System synchronization. The dashed-line rectangle represents the main thread. The rectangles on the left hand side of the main thread form the prefetcher thread. The GUI acts as a separate process, due to the priority queues of TK.

A *select* descriptor holds descriptions of select-list items, and the addresses of output buffers where the names and values of select-list items are stored. A *bind* descriptor holds descriptions of bind variables and indicator variables, and the addresses of input buffers where the names and values of bind variables and indicator variables are stored.

This solution is required in our system due to the fact that the user can dynamically change the dataset to be used during the exploration. Thus, the table which is queried (and implicitly the name and number of the attributes, etc.) changes as well.

# Chapter 7

## Experimental Results

### 7.1 Experimental Inputs

Performance measurements in systems where human interaction is needed require typically some way of simulating the users. The main reasons for that is that exactly the same sequence of operations has to be performed multiple times. User input (also referred to as user script or input script from now onwards) is a sequences of user operations and the time specification of when the operations occur. In XmdvTool, the user input is a sequence of navigation primitives and delays. An example of user script is presented below:

```
1000 3 0.050000
1200 1 0.100000
1400 1 0.150000
1600 2 0.200000
```

The first column contains the time specification, the second the type of the operation and the third the parameters that the operation requires.

The main goal for our input simulation is to provide “data specificity” and “user specificity” to the scripts that we generate. In other words the scripts need to show how particu-

larities in the data may affect the navigation and also how particularities in user behaviour (personal preferences) may influence the same. These particularities provide the hints that we exploit when prefetching.

We simulate the specificity in data by using “hot regions” (Fig. 7.1). Hot regions are places in the navigation space from where some interesting properties in the data are visible and where the user will consequently likely examine. Our assumption is that sooner or later the user will reach those hot regions and thus moving toward them is more probable. The size and the number of these hot regions as well as the probability of using them are factors that can vary as we will show in Section 7.2.

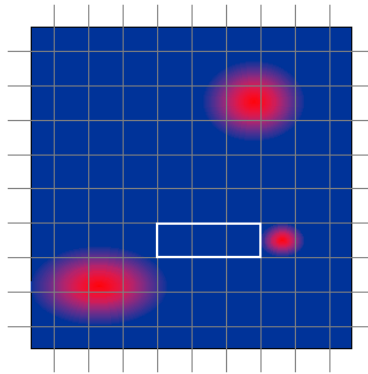


Figure 7.1: Hot regions: selections in the navigation space that provide useful insight into the data.

We also simulate the specificity in the user behaviour. First, we use a probability factor that gives the likelihood that the user keeps moving in the same direction. This seems pretty natural given the navigation environment. Each navigation tool (widget in the GUI) corresponds to a direction. The user will use the same navigation tool for a while and then change to another one, and so on. Second, we distinguish various classes of delays that may occur. We differentiate between short delays that occur between two consecutive events when using the same tool, moderate delays when, for instance, the user changes the navigation tool and large delays when the user analyzes the data. The probability value as well as the delay values can be varied as we will show in Section 7.2.

Given the factors just described, the user moves pseudo-randomly in our experiments, i.e., some of the future possible actions are more probable than others, but choosing among these actions is still performed non-deterministically. The input scripts may affect the system's performance. Therefore, the results we present further are obtained as the average of multiple runs in which the scripts are generated with the same parameters.

The approach used to model user navigation may not be the only possible one. Obviously, we feel that our method generates scripts that are close to real ones; however, further research is needed to validate this assumption.

## 7.2 Settings

All experiments were conducted on an Alpha v4.0 878 DEC station, running Oracle 8.1.5. We used C as the host language and embedded SQL statements for accessing the data in the database.

Throughout all phases of testing we used different types of datasets, both real and synthetic ones, with always consistent results. However, all experiments we report in this thesis are based on synthetic data only. We made this choice to avoid the problems caused by non-homogeneous structures in interpreting the result, since the performance depends to some degree on the shape of the tree, i.e., the distribution of the nodes in the clustering tree.

Thus we used complete trees with a constant fan-out (2). The eleven datasets we used have between 128 (same as IRIS benchmark) and 10 million datapoints, between 8 and 20 dimensions, and between 1,024 ( $2^{10}$ ) and 65,536  $2^{16}$  objects as the maximum number of points displayed at a time. The datasets named: D1, D2, D3, D128, D1k, D2k, D4k, D1k10k, D1k100k, D1k1M, D1k10M, had respectively 4096, 65536, 131072, 128, 1024, 2048, 4096, 10000, 100000, 1000000, and 10000000 objects, and a maximum number of

displayed objects of 2048, 32768, 65536, 128, 1024, 2048, 4096, 1024, 1024, 1024, and 1024 respectively.

We also varied the user input (number of focus regions, delay factor, “keep direction” factor) and some system parameters (prefetching strategy, hints to query optimizer, and size of the data). In all experiments we used navigation scripts containing between 300 and 3000 user operations.

The values we measured during the experiments were: quality, query-based hit ratio, objects-based hit ratio, and latency. The *quality* is the number of objects being displayed during a navigation session. As explained in Chapter 6, the display requests in TK are queued and served when the system is idle. However, if two display requests refer to the same widget, the older one is cancelled. This behaviour may result in lost information, when the display requests are too frequent. This is why we consider the number of objects being displayed as a measure of the visual quality. The hit ratio is the number of hits over the total number of items (queries or objects) requested from the back-end. The latency is the total time, expressed in seconds, in which the user had to wait for her requests to be served, i.e., the on-line loading time.

## 7.3 Experiments

**Experiment 1: MinMax vs. Recursive.** First, we measured the system’s performance when implementing the structure-based brushes using both the MinMax tree technique and a direct implementation of the query specification, i.e., a recursive technique. As SQL does not support recursive views, the recursive technique used an additional attribute to mark the tuples selected by that time by all previous recursion steps. In both cases we used the schema described in Chapter 4 and created simple indices on the join attributes and compound indices on extent and level attributes. Maintaining the indices was not an

issue because the trees were static during our experiments. The results of the experiment are presented in Figures 7.2 – 7.4. As expected, the MinMax method is substantially faster than the recursive one in all cases. These results show clearly that the application of the MinMax technique in our system made visual exploration over large datasets practically feasible, thus accomplishing our ultimate goal.

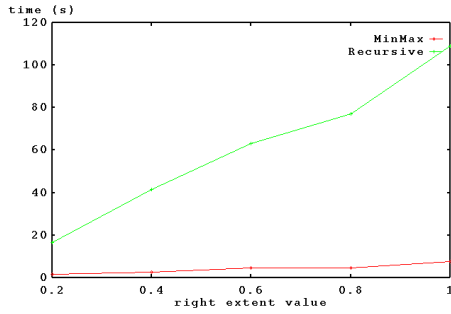


Figure 7.2: MinMax vs. Recursive. Structure-based brushes for dataset D1.

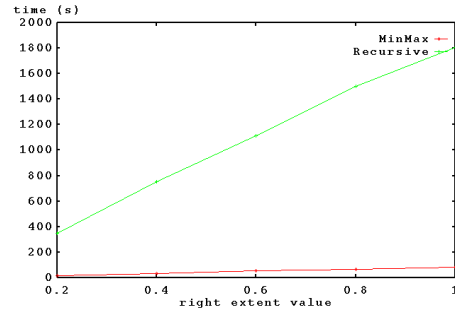


Figure 7.3: MinMax vs. Recursive. Structure-based brushes for dataset D2.

**Experiment 2: Varying brush parameters.** In the next three experiments we analyze the behaviour of the system when the size of the brushes changes. Specifically we vary the level value, the extent values and the size of the dataset. For these experiments we use a compound index on  $(e1, e2, L)$  and a *NOCACHE* hint for the Oracle’s optimizer. Since the brushes are implemented as range queries, we expect that the response time will vary linearly with respect to all these parameters.

**Experiment 2a: Varying the level value.** In this experiment, the extent values are constant (0.25, 0.75) and the level value vary in two-unit increments. As in a complete binary tree the number of points increases exponentially (doubles each level), we expected that the response time would have at most the same growth and thus be linear in the size of the input. However, as one can see in Fig. 7.5, the linear behaviour can be observed only for large levels, while for small ones it is almost constant.

**Experiment 2b: Varying the extent values.** In this experiment, the level value is

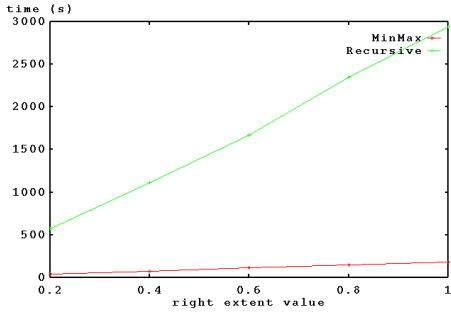


Figure 7.4: MinMax vs. Recursive. Structure-based brushes for dataset D3.

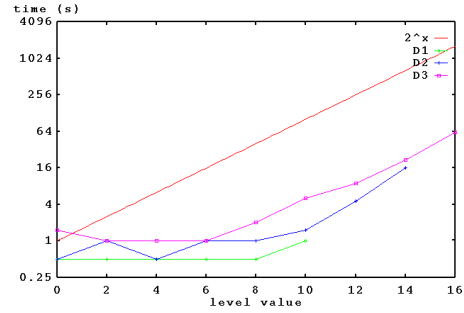


Figure 7.5: Varying the level value. Functions compared against  $2^x$ . Logarithmic y scale.

constant (11 – the leaf level in the smallest dataset) and the extent values varies from (0.0, 0.0) to (0.0, 1.0) in 0.2 increments (captured on the  $x$ -axis). The behaviour is again expected to be linear. This experiment is particularly important since it validates our assumption that the incremental computing of brushes is needed. Indeed, as shown in Fig. 7.6, the increase in the response time is linear. Thus the processing time for a level-based brush is proportionally reduced when computing it incrementally.

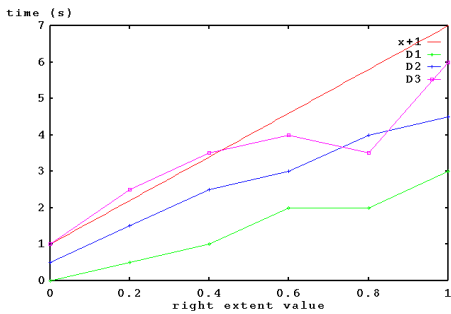


Figure 7.6: Varying the extent value. Functions compared against  $x + 1$ .

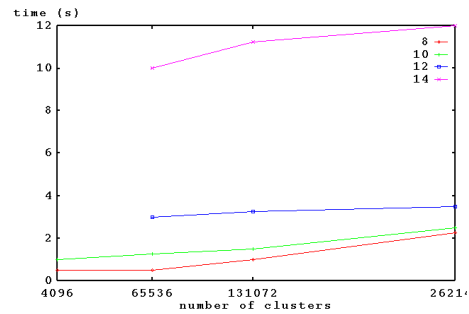


Figure 7.7: Varying the dataset size. Levels 12 and 14 not defined for D1.

**Experiment 2c: Varying the size of the dataset.** In this experiment we used dataset D4, having  $2^{18} = 262144$  tuples. The brush settings are set to values from (0.25, 0.75; 8) to (0.25, 0.75; 14), and the dataset changes from D1 (4096 tuples) to D4 (262144 tuples).

The number of nodes doubled from D1 to D2, from D2 to D3, and from D3 to D4 (shown on the  $x$ -axis). However, as one can see in Fig. 7.7, the resulting time is almost constant. This ensures that our system indeed scales well to large datasets.

**Experiment 3: Varying user input.** In the next three experiments we analyze the behaviour of the system when the user input changes. Specifically we vary the size of the delays between events, the number of the hot regions and the probability parameter that determines if the current direction is abandoned or not. For these experiments we use the dataset D1k100k (having a total of 100,000 objects and 1,000 objects in the maximum active set), and the *NOCACHE* hint for the Oracle’s optimizer. The results represent the measurements for both direction (S2) and focus (S3) prefetching strategies, with minimum buffer size (i.e., 1,000 objects in this case).

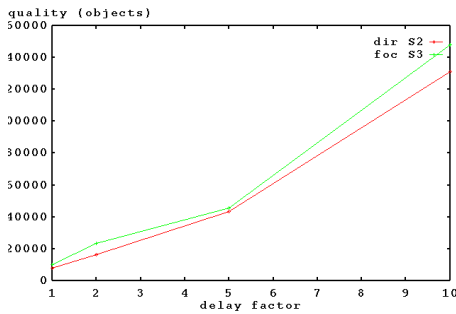


Figure 7.8: Varying the delays for strategy S2 and S3. Measured quality.

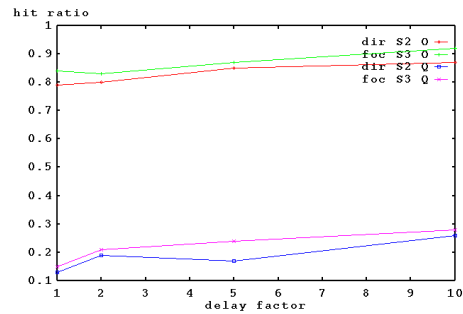


Figure 7.9: Varying the delays for strategy S2 and S3. Measured hit ratio.

**Experiment 3a: Varying the delays.** In this experiment we vary the delay factor. Basically, we multiply the default 1/10–1–5 delays with factors of 1, 2, 5, and 10. Increasing the delays gives the prefetcher more time. Thus, it is natural that the quality (Fig. 7.8), hit ratio (both at the object and at the query level) (Fig. 7.9), and latency (Fig. 7.10) improve. Also, we compare the execution of the direction prefetcher (S2) on multiple data sizes (D128, D1k, D2k, D4k). There is a tendency for the system to perform better than linear as soon as it exceeds the short delay region. This is expected given that for short delays



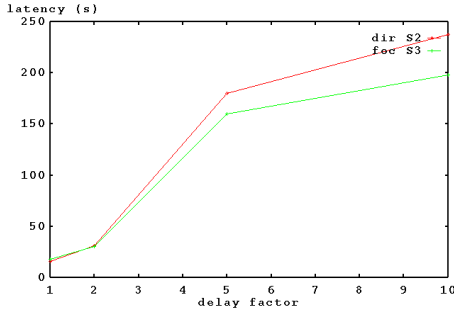


Figure 7.10: Varying the delays for strategy S2 and S3. Measured latency.

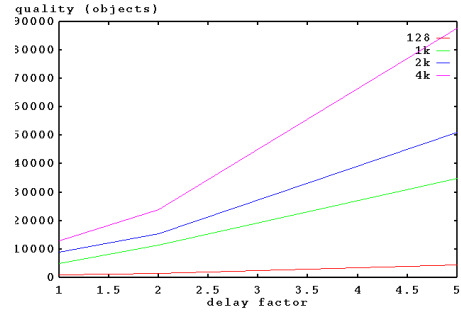


Figure 7.11: Varying the delays for various datasets. Measured quality.

most of the prefetching requests get cancelled.

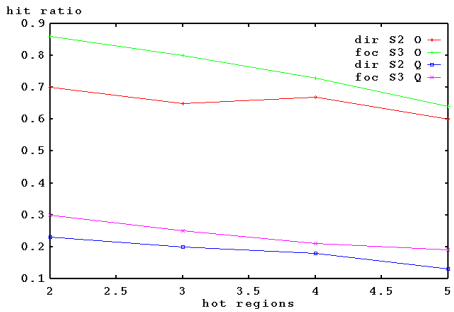


Figure 7.12: Varying the number of hot regions. Measured both query and object hit ratio.

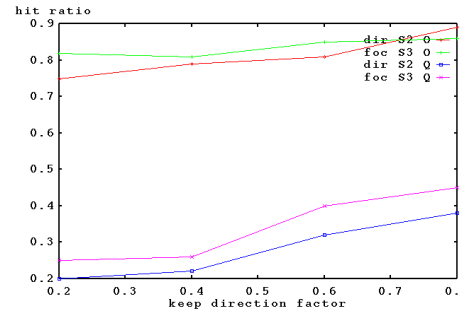


Figure 7.13: Varying the “keep direction” factor. Measured query and object hit ratio.

**Experiment 3b: Varying the hot regions.** In this experiment we vary the number of hot regions from 2 to 5. As we expected, the results show that the performance decreases (Fig. 7.12) when the number of hot regions increases. We present here only the hit ratio measurements, the latency and the quality are not relevant since the input script had to be changed from one measurement to another.

**Experiment 3c: Varying the “keep direction” parameter.** In this experiment we vary the probability of maintaining the same direction when the current sequence of “interruptible” operations is over. Basically, the probability factor gives the likelihood that a

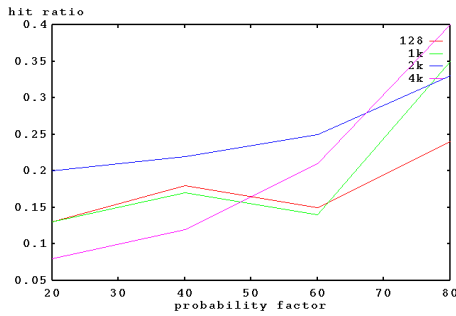


Figure 7.14: Varying the “keep direction” factor for various datasets. Measured the hit ratio.

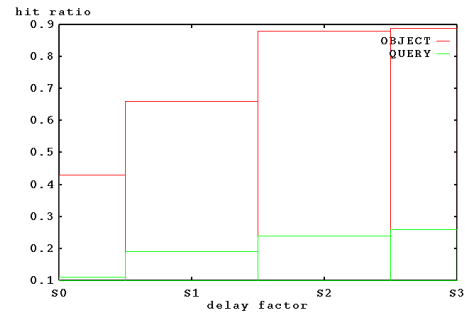


Figure 7.15: Varying prefetching strategy. Measured object and query hit ratio.

user will keep using the same manipulation tool. We use factor values from 0.2 to 0.8, in increments of 0.2. The obvious tendency is that the higher the factor is, the more accurate the prediction is. We again measured the hit ratio (Fig. 7.13); the latency and the quality were irrelevant due to the change of the input scripts. Note that the system uses the same prediction all the time, which is a probability of 0.6. Also, as part of this experiment, we vary the size of the data for the direction (S2) strategy (Fig. 7.14). The same tendency is present.

**Experiment 4: Varying system settings.** In the next three experiments we analyze how, given a “typical” user input (suggested by the previous set of experiments), the performance is influenced by various system settings. We used a user script with three focus regions, a delay multiplication factor of 2 and a “keep direction” factor of 0.60. For these experiments we varied the prefetching strategy, the hints to the query optimizer and the size of the dataset. All of the results were obtained as average of multiple runs (typically 2 to 4), with different input scripts (but generated with the same parameters). The measurements for each point on the graphs is the result of the execution of 500 operation scripts.

**Experiment 4a: Varying the prefetching strategy.** In this experiment we tested the four prefetcher strategies S0, S1, S2, and S3 against two datasets, namely D1k10k and

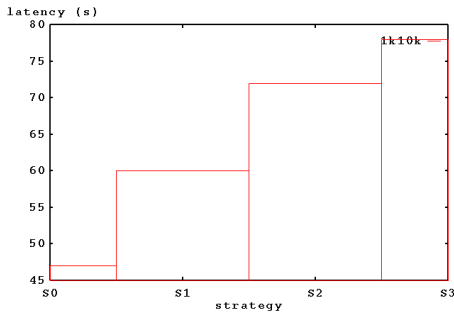


Figure 7.16: Varied prefetching strategy for D1k10k. Measured the latency.

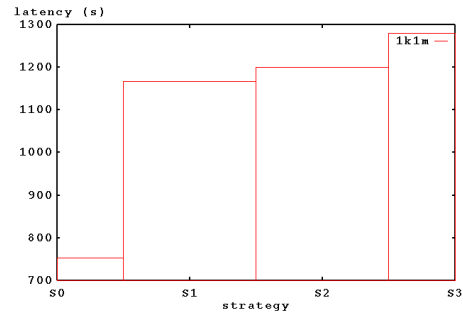


Figure 7.17: Varied prefetching strategy for D1k1M. Measured the latency.

D1k1M. An indirect comparison between S2 and S3 has been already performed in the previous set of experiments. The results confirm that S3 is typically around 10% more efficient than S2 (Fig 7.15). Somehow unexpected, random prefetching (S1) performs close to S2 and S3 for the both datasets (Fig 7.16).

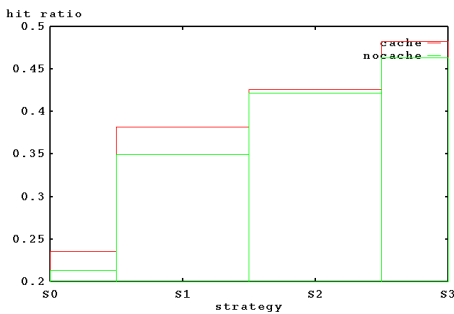


Figure 7.18: Providing hints to the query optimizer. Varying the prefetching strategy. Measured the hit ratio.

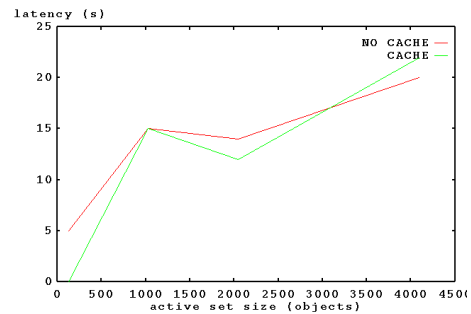


Figure 7.19: Providing hints to the query optimizer. Varying the data size. Measured the latency.

**Experiment 4b: Varying the database cache hints.** In this experiment we evaluate the impact of the database cache policy on the system performance. Since Oracle does not provide support to directly control the cache, we try to vary the degree in which tuples are cached by providing the optimizer with cache hints. The cache hints that we use are “cache” and “nocache”. A “cache” object will persist in the database buffer as long

as possible. A “nocache” object will not be loaded into the buffer. However, nothing prevents the optimizer to read a “nocache” object from cache, if the object is there. Thus, the results of these experiments have a higher degree of deviation. In average however both the hit ratio (Fig 7.18) and the latency (Fig 7.19) improves by about 10%–30%. On the other hand, the gain of using prefetching is usually at least 150%–200%. It follows therefore that our technique has a clear advantage over using a commercial cache only. We varied in this experiment both the prefetch strategy (Fig 7.18) and the data size (Fig 7.19).

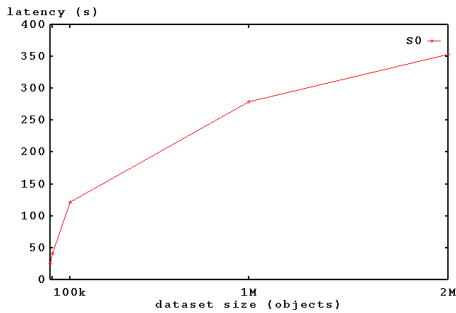


Figure 7.20: Varying the dataset. Measured the hit ratio.

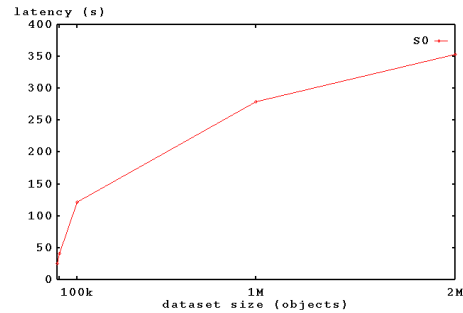


Figure 7.21: Varying the dataset. Measured the latency.

**Experiment 4c: Varying the dataset size.** In this experiment we test the scalability of our method. We use for this purpose the datasets D1k, D1k10k, D1k100k, D1k1M, and D1k2M, and all the prefetch strategies S0, S1, S2, and S3. The buffer size is set to the minimum and the Oracle hints are “nocache”. As one can see, the method scales better than linear (very similar to a logarithmic curve). It follows that the method is very appropriate for exploring large datasets, the goal of this thesis.

**Discussion.** The first set of experiments has demonstrated that recursive processing of queries is far too expensive to be applied in interactive tools such as XmdvTool. Furthermore, the use of MinMax encoding is shown to make the problem tractable. The experiments confirmed again the gain of using pre-computed information to support on-line processing.

The second set of experiments has demonstrated that the processing time for a structure-based brush operation is only proportional to the number of objects being selected and independent from what brush parameter changed. Thus, the time required for computing a sequence of brushes is proportional to the sum of the objects in all these selections. These experiments fully confirmed our approach of computing the structure-based brushes incrementally.

The third set of experiments has analyzed how the user input influences the prefetcher accuracy and thus the system overall efficiency. When there is a clear focus in navigation or there is enough time between events (requests) the performance improves considerably. The experiments confirmed the important role of a good predictor.

The fourth set of experiments has analyzed the performance of the system for a fixed type of user input. The results show that prefetching helps. The system performance improves significantly compared to the case of using a cache policy only, no matter that the cache is at the database side or at the application side. Moreover, we have shown that the system scales better than linearly with respect to the size of the data that is explored.

# Chapter 8

## Conclusions and Future Work

### 8.1 Conclusions

With the increasing amount of data being accumulated nowadays, the need for visually exploring large datasets becomes more and more important. A viable way to achieve scalability in visualization is to integrate visualization applications with database management systems. Such integrations raise however two kind of problems. First, it requires the design of an organization of data and a corresponding query mechanism such that all front-end operations can be translated into efficient database operations. Second, a good memory management strategy should be employed in order to reduce the overhead of database accesses and thus make the use of the database transparent to end-users.

This thesis presents a solution that addresses both problems. The approach is being used to couple XmdvTool4.0, a visualization application for interactive exploration of multivariate data, with an Oracle8i database management system. Experiments for assessing the method showed that, despite the recursive nature of the operations at the interface level, the processing time in our integrated system is only proportional to the number of objects being selected. Moreover, the system scales linearly with respect to

the size of the dataset.

In summary, the main contributions of our approach are:

- The MinMax method was developed in order to improve the applicability of relational dataset systems for a class of new applications including visualization tools or CAD/CAM applications. The method provides important advantages over the traditional recursive approaches when implementing navigation operations on hierarchical structures due to its fast way to compute the ancestors and the descendents of the nodes in the tree.
- We implemented the MinMax technique using Oracle 8 and C as a host language and showed that the MinMax-based queries performed significantly faster than the equivalent recursive queries. We used this technique to implement the query mechanism for XmdvTool [49].
- The caching strategy we employed supports incremental loading of data into the memory buffer. By hashing the objects from the buffer based on their level value and by keeping the objects on each level in order, any query containment test against the data in memory becomes of complexity  $O(1)$ .
- To further reduce the response time in the system, we designed a speculative non-pure prefetcher that brings data into memory when the system is idle. In order to ensure efficiency at interruption, the prefetcher uses a cache replacement policy that combines a low granularity of data (object level) and a “semantic” description of the content of the buffer.
- Users are synthetically generated for testing the system. Navigation patterns simulates data specificity as well as user specificity. Moreover, a proper indirection of the input ensures that exactly the same navigation script can be executed multiple

times.

- Experiments showed that the proposed memory management strategy significantly decreased the latency in the system. The system now scales for datasets of the order  $10^5$ – $10^7$  records.

## 8.2 Future Work

Directions for further research include both refining the current approach and making it more general by dropping some of the constraints that we are enforcing now.

The efficiency in the system could be further improved by developing a more accurate user model. A complex study would need to record and analyze real user input and then produce classes of patterns that invariantly occur in real navigations. We think that our model provides a fair approximation of the users. More research is needed to validate this hypothesis. Once an accurate user model is provided, we could design a more complex predictor, based on that model. If we confidently know how the patterns look, the task of extracting the exact parameters from a real navigation script is a tractable statistical problem.

The system could also be functionally extended by dropping some of the current constraints, as for example the “static” assumption. This assumption has two aspects. First, we might consider dynamic changes of the dataset. It is more and more common to analyze information that suffers intensive updates during the exploration. Second, we might dynamically change the tools that we are using during the exploration itself. Dynamic clustering or dynamic computation of aggregates would be possible for instance.



# Appendix A

## Navigation Operations

### A.1 Notation Conventions

In what follows,  $n_1..n_2$  denote a sequence of integers  $\{n_1, n_1 + 1, \dots, n_2\}$ . A variable  $v$  that takes all the values from  $n_1$  to  $n_2$  will be denoted by  $v = n_1..n_2$ . A variable  $v$  that takes some values from  $n_1$  to  $n_2$  (not necessarily all of them) will be denoted by some  $v \in n_1..n_2$ .

### A.2 Hierarchical Clustering

Let  $X = \{x_1, x_2, \dots, x_m\}$  be a set of base data points. Then,  $P = \{P_1, P_2, \dots, P_n\}$  is defined to be a partition of  $X$  iff:

$$\forall 1 \leq j \leq n: P_j \subseteq X \quad (\text{A.1})$$

$$\forall 1 \leq j_1 \neq j_2 \leq n: P_{j_1} \cap P_{j_2} = \emptyset \quad (\text{A.2})$$

$$\bigcup_{j=1}^n P_j = X \quad (\text{A.3})$$

We denote the set  $X$  on which partition  $P$  is defined by  $\text{support}(P)$ . When using more

than one partition, we indicate the partition number by a superscript, e.g.:  $P^i = \{P_1^i, P_2^i, \dots, P_{n_i}^i\}$ .

Two partitions  $P^{i_1} = \{P_1^{i_1}, P_2^{i_1}, \dots, P_{n_{i_1}}^{i_1}\}$  and  $P^{i_2} = \{P_1^{i_2}, P_2^{i_2}, \dots, P_{n_{i_2}}^{i_2}\}$  are called nested and denoted by  $P^{i_1} \prec P^{i_2}$  ( $P^{i_1}$  nested in  $P^{i_2}$ ), iff:

$$\text{support}(P^{i_1}) = \text{support}(P^{i_2}) \quad (\text{A.4})$$

$$\forall 1 \leq j_2 \leq n_{i_2} : \exists 1 \leq j_1 \leq n_{i_1} : P_{j_2}^{i_2} \subseteq P_{j_1}^{i_1} \quad (\text{A.5})$$

A hierarchical clustering of  $X$  can now be defined as a sequence  $P_X = \{P^1, P^2, \dots, P^k\}$  of partitions of  $X$ , where:

$$P^1 = X \quad (\text{A.6})$$

$$P^k = \{\{x_1\}, \{x_2\}, \dots, \{x_m\}\} \quad (\text{A.7})$$

$$P^1 \prec P^2 \prec \dots \prec P^k \quad (\text{A.8})$$

### A.3 Structure Based Brushes

The hierarchical clustering process, as described in Appendix A.2, results in a tree structure that is formed on the  $P_j^i$  partitions. Property (A.5) above gives us the parent cluster ( $P_{j_1}^{i_1}$ ) for each cluster ( $P_{j_2}^{i_2}$ ) in the tree. In what follows, the parent of a node  $Z$  will be denoted by  $\theta(Z)$ . Moreover, for any set of nodes  $S = \bigcup Z_t$ , we will denote by  $\theta(S)$  the set of the parents of  $S$ . Thus,  $\theta(S) = \bigcup \theta(Z_t)$ .

A structure based brush is basically a set-based function  $SBB : [1..m] \times [1..m] \rightarrow \bigcup_{i=1}^k \bigcup_{j=1}^{n_i} P_j^i$  such that  $SBB(v_1, v_2) = I(v_1, v_2) \cup T(I(v_1, v_2)) \cup \dots \cup T^{k-1}(I(v_1, v_2))$  and where:

1.  $I(v_1, v_2) = \bigcup_{\text{some } j \in 1..n_k} P_j^k \subseteq \bigcup_{j=1}^{n_k} P_j^k$  is the initial selection operator that is applied to the leaf nodes, and

2.  $T(\bigcup_{\text{some } j \in 1..n_i} P_j^i \subseteq \bigcup P_j^i) = \bigcup_{\text{some } j \in 1..n_{i-1}} P_j^{i-1} \subseteq \bigcup P_j^{i-1}$  is a propagation operator that propagates the selection from one level to its adjacent level. ( $T^i$  is the notation for operator  $T$  being applied  $i$  times.)

Any structure-based brush is consequently fully defined by providing a specification for the operators  $I$  and  $T$ .

### A.3.1 The ALL Structure Based Brush

In the ALL structure-based brushes, the  $I$  operator is defined as:

$$I(v_1, v_2) = \{x_{v_1}, \dots, x_{v_2}\} \subseteq \bigcup_{j=1}^{n_k} P_j^k \quad (\text{A.9})$$

The  $T$  operator for an ALL structure-based brush is defined as:

$$T_{ALL}(S = \bigcup_{\text{some } j \in 1..n_i} P_j^i) = \{Z \in \bigcup_{j=1}^{n_i} P_j^{i-1} \mid \neg \exists Y \in (\bigcup_{j=1}^{n_i} P_j^i) \setminus S : Z = \theta(Y)\} \quad (\text{A.10})$$

### A.3.2 The ANY Structure Based Brush

The  $I$  operator for the ANY structure-based brushes is identical with the one for the ALL brushes:

$$I(v_1, v_2) = \{x_{v_1}, \dots, x_{v_2}\} \subseteq \bigcup_{j=1}^{n_k} P_j^k \quad (\text{A.11})$$

The propagation operator  $T$  is however different:

$$T_{ANY}(S = \bigcup_{\text{some } j \in 1..n_i} P_j^i) = \theta(S) \quad (\text{A.12})$$

### A.3.3 The Relational Semantics of Structure-Based Brushes

According to the definitions in Section A.3.1 and Section A.3.2, the propagation operator corresponds to a division and to a join respectively.

Given a relation  $R(x, y, \dots)$ , where  $x$  is a node id and  $y$  is the id of the parent of  $x$ , and considering  $S$  the initial selection defined by the operator  $I$  (same for ALL and ANY structure-based brushes), the ALL and ANY structure-based brushes define the following relational operations:

$$SBB_{ALL} = S \cup (S \div R) \cup (S \div R \div R) \dots = \bigcup_{i=0}^{k-1} S \div \underbrace{R \div \dots \div R}_{i \text{ times}} \quad (\text{A.13})$$

$$SBB_{ANY} = S \cup (S \bowtie R) \cup (S \bowtie R \bowtie R) \dots = \bigcup_{i=0}^{k-1} S \bowtie \underbrace{R \bowtie \dots \bowtie R}_{i \text{ times}} \quad (\text{A.14})$$

# Appendix B

## MinMax Hierarchy Encoding

Let  $\theta(x)$  be the parent of node  $x$ . We define the sets of ancestors  $\text{anc}(x)$  and descendants  $\text{desc}(x)$  of a node  $x$  in the hierarchy as follows:

$$\begin{aligned} \text{anc}(x) = \{y \mid \exists \text{node}_1 = x, \text{node}_2, \dots, \text{node}_{t-1}, \text{node}_t = y : \forall i = 2..t : \\ \text{node}_i = \theta(\text{node}_{i-1})\} \end{aligned} \quad (\text{B.1})$$

$$\begin{aligned} \text{desc}(x) = \{y \mid \exists \text{node}_1 = x, \text{node}_2, \dots, \text{node}_{t-1}, \text{node}_t = y : \forall i = 1..t-1 : \\ \text{node}_i = \theta(\text{node}_{i+1})\} \end{aligned} \quad (\text{B.2})$$

Let now  $I$  be the “initial set” and  $\{(a_1, b_1), (a_2, b_2), \dots, (a_m, b_m)\}$  be the labels of the leaf nodes in that initial set,  $(a_j, b_j) \subseteq I \times I$ ,  $a_1 \leq b_1 \leq a_2 \leq b_2 \leq \dots \leq a_m \leq b_m$  as shown in Section 4. A MinMax tree is a labeled tree of the form  $\text{node}(t) = (a_{\alpha(t)}, b_{\beta(t)})$ ,  $\forall t$ , where:

$$\alpha(t) = \min\{j \mid \text{leaf}_j \in \text{desc}(\text{node}_t)\} \quad (\text{B.3})$$

$$\beta(t) = \max\{j \mid \text{leaf}_j \in \text{desc}(\text{node}_t)\} \quad (\text{B.4})$$

$$\{\text{leaf}_{\alpha(t)}, \dots, \text{leaf}_{\beta(t)}\} \subseteq \text{desc}(\text{node}_t) \quad (\text{B.5})$$

## B.1 Proof of Theorem 1

**Theorem:** Given a MinMax tree  $T$  and two nodes  $x$  and  $y$  of  $T$  whose extent values are  $(a_{x_1}, a_{x_2})$  and  $(a_{y_1}, a_{y_2})$  respectively, node  $x$  is an ancestor of node  $y$  if and only if  $x_1 \leq y_1$  and  $y_2 \leq x_2$ .

**Proof:** We first notice that  $\text{anc}()$  is a monotonic function. Indeed, relation (B.1) implies that if  $x = \theta(y)$  then  $\text{anc}(x) \subseteq \text{anc}(y)$ . Using induction, we further get that:

$$\text{if } x = \text{anc}(y) \text{ then } \text{anc}(x) \subseteq \text{anc}(y). \quad (\text{B.6})$$

Analogously, it can be proven that  $\text{desc}()$  is monotonic, i.e.:

$$\text{if } y = \text{desc}(x) \text{ then } \text{desc}(y) \subseteq \text{desc}(x). \quad (\text{B.7})$$

**Implication  $\Rightarrow$ :** If  $y \in \text{desc}(x)$  then  $x_1 \leq y_1 \leq y_2 \leq x_2$ .

Indeed,  $y \in \text{desc}(x) \Rightarrow_{(\text{B.7})} a_{y_1} \in \text{desc}(y) \subseteq \text{desc}(x) \Rightarrow_{(\text{B.2})} x_1 \leq y_1$ . Similarly,  $y_2 \leq x_2$ . Since  $y_1 \leq y_2$  is true by construction, we have proven that  $x_1 \leq y_1 \leq y_2 \leq x_2$  (q.e.d.).

**Implication  $\Leftarrow$ :** If  $x_1 \leq y_1 \leq y_2 \leq x_2$  then  $y \in \text{desc}(x)$ .

We will prove it by contradiction. Let  $z$  be the lowest level common ancestor of  $x$  and  $y$ . Such a node exists since the root is one common ancestor of  $x$  and  $y$ . It is also unique since two nodes on the same level cannot have common descendents (in a tree hierarchy). Now if  $x = z$  or  $y = z$  then necessarily  $x = y = z$  and  $x_1 = y_1 \leq y_2 = x_2$  (q.e.d.). If  $x \neq z$  and  $y \neq z$  then there exists a child  $c_x = (a_{cx_1}, a_{cx_2})$  of  $z$  such that  $c_x = \text{anc}(x)$  and a child  $c_y = (a_{cy_1}, a_{cy_2})$  of  $z$  such that  $c_y = \text{anc}(y)$ .  $c_x \neq c_y$  follows from the assumption that  $z$  has the lowest level among all the common ancestors. By construction,  $(cx_1, cx_2) \cap (cy_1, cy_2) = \emptyset$ . However,  $c_x = \text{anc}(x)$  implies that  $(x_1, x_2) \subseteq (cx_1, cx_2)$  and  $c_y = \text{anc}(y)$  implies that  $(y_1, y_2) \subseteq (cy_1, cy_2)$ . From the hypothesis  $(y_1, y_2) \subseteq (x_1, x_2)$  and therefore

$(cx_1, cx_2) \cap (cy_1, cy_2) \supseteq (y_1, y_2) \neq \emptyset$  (contradiction) (q.e.d.).

## B.2 Proof of Theorem 2

**Theorem:** Given the brush values  $v_{min}$  and  $v_{max}$ , an ALL structure-based brush generates the union of all nodes  $x = (a_{x_1}, a_{x_2})$  whose extents are fully contained in the brush interval  $(v_{min}, v_{max})$ , i.e.,  $(x_1, x_2) \subseteq (v_{min}, v_{max})$ .

**Proof:** Let  $x = (a_{x_1}, a_{x_2})$  be a node and  $SBB = SBB_{ALL}(v_1, v_2)$  be an ALL structure-based brush.

**Implication  $\subseteq$ :** If  $x \in SBB$  then  $(x_1, x_2) \subseteq (v_1, v_2)$ .

$x \in SBB \Rightarrow \exists 0 \leq i \leq k : x \in T^i(I(v_1, v_2)) \Rightarrow_{(A.10)} \neg \exists y \notin T^{i-1}(I(v_1, v_2)) : \theta(y) = x \Rightarrow$   
 $p^{-1}(x) \in T^{i-1}(I(v_1, v_2)) \Rightarrow \dots \Rightarrow (p^{-1})^i(x) \in T^0(I(v_1, v_2)) = I(v_1, v_2) \Rightarrow a_{x_1} \in I(v_1, v_2) \ \& \ a_{x_2} \in$   
 $I(v_1, v_2) \Rightarrow x_1 \in (v_1, v_2) \ \& \ x_2 \in (v_1, v_2) \Rightarrow v_1 \leq x_1 \leq x_2 \leq v_2 \Rightarrow (x_1, x_2) \subseteq (v_1, v_2)$  (q.e.d.).

**Implication  $\supseteq$ :** If  $(x_1, x_2) \subseteq (v_1, v_2)$  then  $x \in SBB$ .

We will prove it by contradiction. If  $x = P_j^i$  then:  $x \notin SBB \Rightarrow \exists x^{i+1} \in \bigcup_{j=1}^{n_{i+1}} P_j^{i+1} \setminus$   
 $T^{k-i-1}(I(v_1, v_2)) : x = \theta(x^{i+1}) \Rightarrow \exists x^{i+2} \in \bigcup_{j=1}^{n_{i+2}} P_j^{i+2} \setminus T^{k-i-2}(I(v_1, v_2)) : x^{i+1} = \theta(x^{i+2}) \Rightarrow$   
 $\dots \Rightarrow \exists x^k \in \bigcup_{j=1}^{n_k} P_j^k \setminus I(v_1, v_2) : x^{k-1} = \theta(x^k) \Rightarrow \exists x^k = (a_{a_1}, a_{a_2}) \in \text{desc}(x) : x^k \notin I(v_1, v_2) \Rightarrow_{(T1)}$   
 $\exists (a_1, a_2) \subseteq (x_1, x_2) : (a_1, a_2) \not\subseteq (v_1, v_2) \Rightarrow (x_1, x_2) \not\subseteq (v_1, v_2)$  (contradiction) (q.e.d.).

## B.3 Proof of Theorem 3

**Theorem:** Given the brush values  $v_{min}$  and  $v_{max}$ , an ANY structure-based brush generates all the nodes  $x = (a_{x_1}, a_{x_2})$  whose intersection with the brush interval  $(v_{min}, v_{max})$  is not empty, i.e.,  $(x_1, x_2) \cap (v_{min}, v_{max}) \neq \emptyset$ .

**Proof:** Let  $x = (a_{x_1}, a_{x_2})$  be a node and  $SBB = SBB_{ANY}(v_1, v_2)$  be an ANY structure-based brush.

**Implication  $\subseteq$ :** If  $x \in SBB$  then  $(x_1, x_2) \cap (v_1, v_2) \neq \emptyset$ .

$x \in SBB \Rightarrow \exists 0 \leq i \leq k : x \in T^i(I(v_1, v_2)) \Rightarrow \exists x^{k-i} = x \in \mathbf{desc}(x) : x \in T^i(I(v_1, v_2)) \Rightarrow \exists x^{k-i+1} \in \mathbf{desc}(x), x \in p^{-1}(x^{k-i}) : x^{i+1} \in T^{i-1}(I(v_1, v_2)) \Rightarrow \dots \Rightarrow \exists x^k \in \mathbf{desc}(x), x \in p^{-1}(x^{k-1}) : x^k = (a_{a_1}, a_{a_2}) \in I(v_1, v_2) \Rightarrow_{(A.11)} v_1 \leq a_1 \leq a_2 \leq v_2$ . But,  $x^k = (a_{a_1}, a_{a_2}) \in \mathbf{desc}(x) \Rightarrow_{(T1)} x_1 \leq a_1 \leq a_2 \leq x_2 \Rightarrow (x_1, x_2) \cap (v_1, v_2) \supseteq (a_1, a_2) \neq \emptyset$  (q.e.d.).

**Implication  $\supseteq$ :** If  $(x_1, x_2) \cap (v_1, v_2) \neq \emptyset$  then  $x \in SBB$ .

$(x_1, x_2) \cap (v_1, v_2) \neq \emptyset \Rightarrow \exists t : t \in (x_1, x_2) \cap (v_1, v_2) \equiv t \in (x_1, x_2) \ \& \ t \in (v_1, v_2)$ .

$t \in (x_1, x_2) \Rightarrow a_t \in \mathbf{desc}(x) \Rightarrow \exists$  a path  $\{a_t = x^k, x^{k-1}, \dots, x^i = x\}$  from  $a_t$  to  $x$  such that  $x^{k-1} = \theta(x^k), x^{k-2} = \theta(x^{k-1}), \dots, x^i = \theta(x^{i-1})$ .

$t \in (v_1, v_2) \Rightarrow a_t \in I(v_1, v_2) \Rightarrow x^k \in I(v_1, v_2) \Rightarrow_{(A.12)} x^{k-1} \in T(I(v_1, v_2)) \Rightarrow_{(A.12)} x^{k-2} \in T^2(I(v_1, v_2)) \Rightarrow_{(A.12)} \dots \Rightarrow_{(A.12)} x^i \in T^{k-i}(I(v_1, v_2)) \Rightarrow x \in T^{k-i}(I(v_1, v_2)) \Rightarrow x \in SBB$  (q.e.d.).



# Appendix C

## Complexity of Memory Operations

For easy reference, buffer operations, as introduced in Chapter 5, are listed again here:

- A: **Remove old objects.** Get the objects with the lowest probability that reside in the buffer (and further remove them one at a time when more room in the buffer is needed).
- B: **Bring new objects.** Place an object from the cursor buffer into the memory buffer (and rehash the buffer entry).
- C: **Display active set.** Get those objects from the buffer that form the active set (and send them to the graphical interface to have them displayed).
- D: **Recompute probabilities.** Recompute the probabilities of the objects in the buffer once the active window gets changed (to ensure accurate predictions in the future).
- E: **Test containment.** Test whether the new active set fully resides in the buffer and get the missing objects (if any) from the support set (when a new request is issued).

## C.1 Full Size Probability Table

In what follows, the complexity of buffer access operations is presented. The function that we measured was the number of buffer accesses, i.e., the number of buffer entries that are visited. For a better understanding, a full version of the probability table is assumed first.

- *Remove old objects*

**Complexity:** There are between  $n/2$  and  $\log n$  buffer accesses, in average, for the first element of a single bucket and exactly one buffer access for the following elements in the bucket list. Since the lists are usually of order  $n$ , it follows that operation A is  $O(1)$  with a good probability.

- *Bring new objects*

**Complexity:** There is one operation and a half, on average, for level based rehashing and as many as  $n/2$  operations in the average for the worst distribution for the probability based rehashing. The operation is therefore  $O(n)$ .

- *Display active set*

**Complexity:** There are  $n$  buffer accesses in the worst case, but there are no unnecessary ones; so that this is optimal. The operation is  $O(n)$ .

- *Recompute probabilities*

**Complexity:** The objects preserve their level value so no change of the level lists is needed. However, the probability table needs to be rebuilt, and this takes  $n + n^2/2$  operations:  $n$  is for deleting the lists (this can be done together with the probability recomputation step) and  $n^2/2$  for creating the new ones (it is basically one list insertion for each object). The operation is  $O(n^2)$ .

- *Test containment*

**Complexity:** Testing is  $O(1)$ , loading the next  $m$  objects requires  $m * |A| * |B|$  buffer accesses, where  $|X|$  is the number of buffer accesses for operation  $X$ .

## C.2 Reduced Probability Table

We now consider the case of a reduced (simplified) probability table. We measure again the number of buffer accesses. As we shall see, the complexity of some operations decreases.

- *Remove old objects*

**Complexity:** Same as before.

- *Bring new objects*

**Complexity:** There are on average three and a half buffer accesses for rehashing in this case. The operation is therefore  $O(1)$ .

- *Display active set*

**Complexity:** Same as before.

- *Recompute probabilities*

**Complexity:** There are  $n + 2m$  buffer accesses. Since  $m$  is constant, the operation is  $O(n)$ .

- *Test containment*

**Complexity:** Same as before.

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